Modeling and Analysis of the Interaction between Opinions and Actions among Heterogeneous Agents

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***Abstract*—In complex networks where agents exchange opinions and influence each other's decisions, their interactions and decisions are shaped by many personal and social factors, such as bounded rationality and agent heterogeneity. In addition, the discrepancy between opinions and actions, known as the value-action gap, significantly impacts agents' decisions. To better model and analyze the dynamics among heterogeneous agents' opinions and behaviors, in this paper, we propose an opinion-action co-evolution model that captures the intertwined interplay between individual opinions and decision-making process. Our model accounts for agents' bounded rationality by considering both limited cognitive ability and non-cognitive subjective interference. We incorporate a cost function based on behaviour reasoning theory to model the value-action gap. We theoretically analyze the steady states of opinion-action co-evolution for all agent types and use simulations to validate our theoretical analysis and gain further insights.**

***Index Terms*—Co-evolution, opinion, action, heterogeneity**

I. Introduction

In complex networks with intricate structure involving diverse agents and interconnected relationships, agents interact dynamically, leading to changes in their attributes and overall characteristics. The dynamics of opinions and actions in these networks have received lots of attentions in various scenarios, from everyday transactions and cooperations to large-scale events such as elections. According to Cambridge Dictionary, an opinion is a thought or belief about something or someone, while an action refers to something done or performed. They do not exist in isolation as analyzed in most existing works. Rather, they are intertwined in a complex and interrelated manner that influences agents' decision-making processes, social norms, and collective behaviors [1]. Studying the complicated dynamics of opinions and actions is critical to understand and predict complex social behaviors and outcomes, addressing critical issues such as polarization, public health compliance, and collective decision-making.

Numerous factors influence the interplay and evolution of opinion and action. Bounded rationality, which suggests that people tend to find adequate solutions rather than optimal solutions due to various limitations [2], is a significant personal factor in this dynamics, providing a more realistic representation of human decision processes. Beyond personal factors, social influence is essential, as agents within a network often affect each other. Moreover, agents exhibit heterogeneity due to differences in statuses, functions and objectives, e.g., publishers and their audience, suppliers and consumers, or candidates and voters. So the mechanisms by which they update their opinions and actions are different, and social influences occur both within homogeneous groups and across heterogeneous types of agents. Additionally, a value-action gap, also known as the attitude-behavior gap, often exists during the co-evolution of opinion and action. This gap refers to the discrepancy between individuals' stated opinions and their actions [3]. For instance, despite preferences for renewable energies, their adoption is slow due to additional costs and other factors [4].

In summary, it is crucial to understand the complicated interaction between opinion dynamics and action change among heterogeneous agents, and model and analyze factors that influence the complex co-evolution of opinion and action.

A. Literature Review

In the following, we review recent works in the evolution of opinion and action, bounded rationality, and value-action gap.

1) Opinion and Action Evolution: Numerous studies have explored separately on opinion dynamics [5] and action dynamics [6]. Research in social psychology has shown that opinion dynamics and behavioral decisions are deeply intertwined [1]. In other words, an agent's opinion plays a key role in his/her decision-making process, and observed behaviors of others can, in turn, shape his/her opinion. However, only a few works have investigated their co-evolution, which can be categorized into two types: those considering one-sided influence and those considering mutual effects. The former type, such as the seminal CODA model [7], generally regards opinions as continuous variables and models their direct impact on discrete actions. The latter type, such as the study by Luo et al. on the co-evolution of opinion and action in advertising contexts, considers mutual effects [8]. Subsequent works have provided viable frameworks and theoretical analyses, such as conditions for convergence, consensus, and stable states [9], [10]. Authors in [11] explored the co-evolution of actions and opinions on a two-layer network using potential game theory. However, all existing studies ignore the heterogeneity of agents and individual features such as bounded rationality and the value-action gap.

2) Bounded Rationality: Compared with previous theories of unbounded rationality, the bounded rationality emphasizes

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the limits of agents' ability to make decisions given specific conditions and constraints, thus agents generally can not seek the optimal solution when achieving goals [12]. According to existing works, three kinds of constraints explain this "boundedness": limitations of cognitive ability, interference of non-cognitive subjective factors, and environment conditions [13]. The first type is embodied in the agent's limited ability to get and process information in the face of complexity and uncertainty [14]. The second type reflects the influence of factors such as emotion, motivation and social psychology. For instance, Simon proposed aspiration level and rules capturing subjective emotional responses and motivation adjustments in [15]; aspiration adaptation theory contains similar ideas [16]; and prospect theory models decision-making under risk, emphasizing that people's aversion to loss is much greater than the pleasure brought by gain [17]. However, existing works typically focus on certain individual characteristics in bounded rationality, without simultaneously addressing and analyzing phenomena like the value-action gap.

3) Value-action Gap: The value-action gap is primarily studied in environmental and social psychology [4], [18]. The causes of this gap can be broadly categorized into environmental influences that focus on social values and moral norms [19], and personal factors that are based on self-interest and individual choice [20]. Personal factors are generally considered to be the primary reasons, such as additional costs, sufficient benefits, and alignment with personal values. Several models have been proposed to explain the value-action gap, including the Theory of Planned Behavior (TPB) [21] and Behaviour Reasoning Theory (BRT) [22]. Considering the reasons for and against the products, Claudy et al. applied BRT to provide a new paradigm for explaining the relationship between consumers' attitudes, values and adoption intentions [4]. However, existing research only validates models through causal analysis and lacks quantitative models and corresponding analyses.

B. Our Contribution

In this work, we jointly consider personal factors and social influences in the complex interaction of agents' opinions and actions, and propose a unified framework to study their coevolving process. The main contributions of this paper are:

- We propose a co-evolution model for opinions and actions among heterogeneous agents including dominant agents and normal agents, to model the complicated interaction among different agents as well as their opinion dynamics and behaviors.
- We mathematically model the bounded rationality during agent's decision-making process and their heterogeneity, to address personal factors and social influences in the dynamic change of opinions and actions respectively. Furthermore, we consider the value-action gap to capture the discrepancy between interactive opinions and actions.
- We theoretically analyze the opinion-action co-evolution for two types of agents respectively, and derive its steady state. This investigation provides important insights into the long-term behavior patterns.

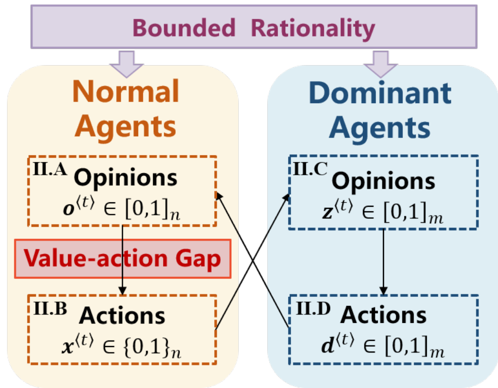


Fig. 1: The model of opinion-action co-evolution for heterogeneous agents.

II. The Opinion-Action Co-evolution Model

In this section, we model the co-evolution of opinions and actions among heterogeneous agents, considering their bounded rationality, social influence and value-action gap. Using continual transactions of products between fixed suppliers and consumers as an example, suppliers initiate product offerings and persuade consumers to make purchases. These agents are heterogeneous due to their differing statuses and objectives. Suppliers' opinions are their emphasis on the product, and their actions reflect the efforts and resources (e.g., advertising intensity and manpower) they invest in encouraging purchases. Consumers' opinions are their thoughts about the product and the supplier, while their actions represent their purchasing choices. As interactions continue, agents adjust their opinions and decisions accordingly. We use a directed graph to model the social connections in complex networks among different agents.

To distinguish heterogeneous agents with various status and objectives, we divide all agents into two categories: dominant agents and normal agents, corresponding to suppliers and consumers in the above example, respectively. Dominant agents are initiators or service providers, aiming to make the audience, i.e., normal agents, approve their objectives and take actions. Normal agents, the targets of dominant agents, primarily focus on their own needs. Empirical evidence and models incorporating the bounded rationality have investigated the heterogeneity of agents [23]. Specifically, some sophisticated agents can process information with better abilities and fewer constraints, aligning with dominant agents; while others have more limited capabilities, corresponding to normal agents. Moreover, research on the value-action gap is typically conducted among target group of specific policy or products, indicating that this gap generally occurs among normal agents [4]. Therefore, we introduce different update mechanisms for the decision-making processes of dominant and normal agents.

The co-evolution model is illustrated in Fig. 1. For two types of agents, they both have two interactive attributes: opinions and actions. Here we consider a simplified scenario that the action of an agent is only influenced by his/her opinion, and the opinion would be affected by the observed actions of other type of agents. The value-action gap exists when normal agents take actions based on their own opinions. When different types of agents decide on their opinions and actions, bounded rationality plays a role to different degree, thus the four submodels in Fig. 1, i.e., II.A, II.B, II.C and II.D that introduced in the following, operate in different ways.

Assume there are n normal agents and m dominant agents. We divide time into slots, and in each time slot t, every agent updates his/her opinion and action based on the observed opinions and actions of others and his/her own interest. Normal agents' continuous opinion vector and discrete action vector are denoted by $\boldsymbol{o}^{\langle t\rangle} \in [0,1]_n$ and $\boldsymbol{x}^{\langle t\rangle} \in \{0,1\}_n$ respectively. Dominant agents' continuous opinion vector and action vector are denoted by $\boldsymbol{z}^{\langle t\rangle} \in [0,1]_m$ and $\boldsymbol{d}^{\langle t\rangle} \in [0,1]_m$ respectively. Here, a value of 1 in continuous opinion variables signifies strong positivity, while 0 denotes strong negativity. For discrete action variables for normal agents, 1 indicates execution of behaviors such as purchase or vote, and 0 indicates the opposite. For continuous action variables for dominant agents, 1 means maximal effort towards action, while 0 means inaction. We assume that different types of agents can only observe others' actions. Then we introduce submodels of opinions and actions for normal agents and dominant agents respectively.

A. Update Mechanisms for Normal Agents' opinions

We define the update mechanism of normal agents' opinion vector $\boldsymbol{o}^{\langle t\rangle}$ based on Friedkin-Johnsen (FJ) model [24] as:

$$\boldsymbol{o}^{\langle t\rangle} = \boldsymbol{A}_n \boldsymbol{W} \boldsymbol{o}^{\langle t-1\rangle} + (\boldsymbol{I} - \boldsymbol{A}_n)\boldsymbol{s}^{\langle t\rangle}, \tag{1}$$

where $\boldsymbol{W}$ is the graph weight matrix with $w_{i,j} \geq 0$ and $\sum_{j=1}^{n} w_{i,j} = 1$ and represents the influence of connected normal neighbors' opinions. $\boldsymbol{A}_n = \mathrm{diag}(a_1, \cdots, a_n)$ is an $n \times n$ diagonal susceptibility matrix of normal agents indicating their sensitivity to social influences. $\boldsymbol{s}^{\langle t\rangle}$ is normal agents' internal opinion vector, which represents individual current thoughts.

The internal opinion of a normal agent is affected by the observed actions of dominant agents, the agent's initial opinion and his/her previous internal opinion. First, when modeling the impact of observed actions of dominant agents, we consider agents' bounded rationality caused by limited cognitive ability. We assume that normal agents cannot ascertain the true values of dominant agents' actions and have to estimate them based on limited information. Thus, similar to [25], we define $\hat{d}_{ij}^{\langle t-1\rangle} \sim \mathrm{TN}\left(d_j^{\langle t-1\rangle}, \sigma_i^2, 0, 1\right)$ as the estimate of dominant agent j's action generated by normal agent i in time slot $t-1$, sampled from a truncated normal distribution on $[0,1]$, with the actual action value $d_j^{\langle t-1\rangle}$ as the mean and σ_i^2 as the variance, reflecting the diversity in perception. The normal agent i forms an overall impression of all dominant agents' actions via the mean $\overline{\hat{d}_i^{\langle t-1\rangle}} = \frac{1}{m}\sum_{j=1}^{m} \hat{d}_{ij}^{\langle t-1\rangle}$. We then represent the estimate vector of dominant agents' actions generated by all normal agents by $\overline{\hat{\boldsymbol{d}}^{\langle t-1\rangle}} = \left[\overline{\hat{d}_1^{\langle t-1\rangle}}, \overline{\hat{d}_2^{\langle t-1\rangle}}, \cdots, \overline{\hat{d}_n^{\langle t-1\rangle}}\right]^T$. The influence of dominant agents' actions on normal agents is captured by $\boldsymbol{F}_n = \mathrm{diag}(F_{n1}, \cdots, F_{nn})$, which is an $n \times n$ diagonal influence matrix of normal agents representing the susceptibilities to the actions of dominant agents.

The initial opinion vector of normal agents, denoted by $\boldsymbol{s}^{\langle 0\rangle}$, is a crucial factor in opinion dynamics as stated in Friedkin-Johnsen (FJ) model [24]. The influence degree is represented by $\boldsymbol{\Gamma} = \mathrm{diag}(\gamma_1, \cdots, \gamma_n)$, an $n \times n$ diagonal stubbornness matrix of normal agents representing their stubbornness for initial opinions.

Then following prior works in [26], we model the internal opinion vector of normal agents $\boldsymbol{s}^{\langle t\rangle}$ as:

$$\boldsymbol{s}^{\langle t\rangle} = \boldsymbol{F}_n \overline{\hat{\boldsymbol{d}}^{\langle t-1\rangle}} + \boldsymbol{\Gamma}\boldsymbol{s}^{\langle 0\rangle} + (\boldsymbol{I} - \boldsymbol{F}_n - \boldsymbol{\Gamma})\,\boldsymbol{s}^{\langle t-1\rangle}. \tag{2}$$

B. Update Mechanisms for Normal Agents' actions

We assume that normal agents' actions are only affected by their individual opinions, value-action gap and their bounded rationality, without considering the influence of neighbors' actions. Following [27], we assume that normal nodes' actions evolve according to a stochastic process. Specifically, we formulate the probability for agent i to adopt action $x \in \{0,1\}$ at time t based on the mixed logit model [28], considering agents' uncertainty in determining strategy. It is defined as:

$$P\left(x_i^{\langle t\rangle} = x\right) = \frac{e^{\pi_i(x)}}{e^{\pi_i(x)} + e^{\pi_i(1-x)}}, \tag{3}$$

where $\pi_i(x)$ is the payoff when the normal agent i adopting action x given his/her current opinion $o_i^{\langle t\rangle}$. When defining the payoff function, we consider issues including the agent's opinion $o_i^{\langle t\rangle}$, the cost $C_n\left(x, o_i^{\langle t\rangle}\right)$ modeling the value-action gap, and individual bounded rationality represented by the value function $V_n(\cdot)$:

$$\pi_i(x) = V_n\left(\kappa\left[\lambda_i o_i^{\langle t\rangle} + (1-\lambda_i)\, C_n\left(x, o_i^{\langle t\rangle}\right)\right]\right). \tag{4}$$

In (4), $\kappa > 1$ is the expansion coefficient, and $\lambda_i \in [0,1]$ is the trade-off coefficient between individual opinion and the cost.

$V_n(\cdot)$ is the value function in the Prospect Theory [17]:

$$V_n(x) = \begin{cases} x^{\alpha}, & x \geq 0, \\ -\rho(-x)^{\beta}, & x < 0. \end{cases} \tag{5}$$

It means that subjective value deviates from actual value due to cognitive biases and subjective evaluations under risk. Agents exhibit risk aversion and tend to underestimate positive values, while showing risk preference and overestimating negative values. In addition, the sensitivity to negative values is stronger than to positive values. This embodies agents' bounded rationality due to non-cognitive subjective factors. From existing studies [17], we set the parameters as $\alpha = \beta = 0.88, \rho = 2.25$.

$C_n\left(x, o_i^{\langle t\rangle}\right)$, whose value is negative, quantifies the cost of adopting the action x with opinion $o_i^{\langle t\rangle}$, and is designed based on BRT to explain the value-action gap [4]. It represents the price the agent must pay, which discourages the agent from taking the action, and is denoted by:

$$C_n\left(x, o_i^{\langle t\rangle}\right) = \begin{cases} o_i^{\langle t\rangle} - 1 - c, & x = 1, \\ -o_i^{\langle t\rangle}, & x = 0, \end{cases} \tag{6}$$

where $c > 0$ is the additional loss when $x = 1$, as the execution generally costs more. We can see that the absolute value of cost function is the absolute difference between the opinion and action as well as the additional loss, and the cost is lower when the values of opinion and action are closer.

C. Update Mechanisms for Dominant Agents' Opinions

During the interaction, opinions of dominant agents are primarily influenced by the actions of normal agents. Then the opinion vector of dominant agents $\boldsymbol{z}^{\langle t\rangle}$ is updated by:

$$\boldsymbol{z}^{\langle t\rangle} = \boldsymbol{F}_d\, \boldsymbol{\omega}\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big) + (\boldsymbol{I}-\boldsymbol{F}_d)\boldsymbol{z}^{\langle t-1\rangle}, \tag{7}$$

where $\boldsymbol{F}_d = \mathrm{diag}\left(F_{d1}, \cdots, F_{dm}\right)$ is an $m \times m$ diagonal influence matrix of dominant agents representing their susceptibilities to the influence of normal agents' actions.

In (7), $p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}$ is the proportion of normal agents adopting the action 1 in the time slot $t-1$. $\boldsymbol{\omega}\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big) = \Big[\omega_1\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big), \cdots, \omega_m\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big)\Big] : [0,1] \rightarrow [0,1]^{m\times 1}$ is a mapping vector function of $p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}$, signifying the effect of normal agents' actions on different dominant agents. When defining the mapping function $\boldsymbol{\omega}\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big)$, note that based on empirical studies, estimates of the frequency of events by human observers are typically distorted, with the relative low frequency overestimated and the relative high frequency underestimated. Therefore, following the work in [29], the distortion of the proportion $\omega_i\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big)$ is denoted as the form of linear transformations of the log odds:

$$\mathrm{Lo}\Big(\omega_i\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big)\Big) = \epsilon_i\, \mathrm{Lo}\Big(p_{\boldsymbol{x}=\boldsymbol{1}}^{\langle t-1\rangle}\Big) + (1-\epsilon_i)\mathrm{Lo}(p_0). \tag{8}$$

The log odds $\mathrm{Lo}(p) = \log \frac{p}{1-p}$. $\epsilon_i \in (0,1)$ is the slope of the linear transformation, and p_0 is the fixed point which means that the value of p is mapped to itself. Note that according to [29], the prediction of fixed point p_0 in (8) is the inverse of the number of classes, so here p_0 is set to be 0.5.

D. Update Mechanisms for Dominant Agents' Actions

We adopt the continuous generation model to formulate the actions of dominant agents in [30]. In time slot t, the dominant agent i's action is affected by all his/her previous and current opinions, which can be understood as a process of continuous revision on current action decision from the influence of opinion. He/she will decide on the action by random sampling from the following beta distribution:

$$d_i^{\langle t\rangle} \sim \mathrm{Beta}\left(\sum_{l=0}^{t} z_i^{\langle l\rangle}, t+1-\sum_{l=0}^{t} z_i^{\langle l\rangle}\right). \tag{9}$$

The probability density function of $\mathrm{Beta}\,(a,b)$ is $f(x;a,b) = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}x^{a-1}(1-x)^{b-1}$, and $a = \sum_{l=0}^{t} z_i^{\langle l\rangle}$ and $b = t+1-\sum_{l=0}^{t} z_i^{\langle l\rangle}$ in (9).

III. The Analysis of Opinion and Action Dynamics and Steady State

Based on the proposed co-evolution model, we analyze how both normal and dominant agents' opinions and actions change dynamically, and obtain theoretical results of the evolutionary steady states. Here we give conclusions directly and the detailed derivation can be found in Supplementary [31].

A. Opinion Dynamics

The dynamic results of normal agents' and dominant agents' opinions can be represented by iterations according to their evolution rule in (1), (2) and (7) as:

$$\boldsymbol{o}^{\langle t\rangle} = (\boldsymbol{A}_n\boldsymbol{W})^{t-1}\boldsymbol{s}^{\langle 0\rangle} + \sum_{i=0}^{t-2}(\boldsymbol{A}_n\boldsymbol{W})^{t-i}(\boldsymbol{I}-\boldsymbol{A}_n)\boldsymbol{s}^{\langle i\rangle}, \tag{10}$$

$$\boldsymbol{s}^{\langle t\rangle} = \left[\sum_{i=0}^{t-2}\left(\boldsymbol{I}-\boldsymbol{F}_n-\boldsymbol{\Gamma}\right)^i\boldsymbol{\Gamma} + \left(\boldsymbol{I}-\boldsymbol{F}_n-\boldsymbol{\Gamma}\right)^{t-1}\right]\boldsymbol{s}^{\langle 0\rangle} + \sum_{i=0}^{t-2}\left(\boldsymbol{I}-\boldsymbol{F}_n-\boldsymbol{\Gamma}\right)^i\boldsymbol{F}_n\overline{\hat{\boldsymbol{d}}^{\langle t-1-i\rangle}}, \tag{11}$$

$$\boldsymbol{z}^{\langle t\rangle} = \sum_{i=0}^{t-1}(\boldsymbol{I}-\boldsymbol{F}_d)^i\,\boldsymbol{F}_d\;\boldsymbol{\omega}\Big(p_{\boldsymbol{x}=1}^{\langle t-1-i\rangle}\Big) + (\boldsymbol{I}-\boldsymbol{F}_d)^t\boldsymbol{z}^{\langle 0\rangle}. \tag{12}$$

where $\boldsymbol{s}^{\langle i\rangle}$ in (10) is substituted by (11). Thus when the initial opinion vector $\boldsymbol{s}^{\langle 0\rangle}, \boldsymbol{z}^{\langle 0\rangle}$, the vector of observed dominant agents' action $\overline{\hat{\boldsymbol{d}}^{\langle t\rangle}}$ and the proportion of normal agents' actions $p_{\boldsymbol{x}=1}^{\langle t\rangle}$ in each time slot are known, the opinions of both types of agents in current time slot can be directly obtained.

B. The Steady State

Note that when the actions of normal agents evolve to a steady state, although some agents would change their actions with probabilities, the proportion of normal agents who take actions 1 will remain asymptotically stable. Therefore, we pay attention to the proportion of agents adopting the action 1 as the steady state of normal agents' actions. We can show that:

$$\mathbb{E}[\boldsymbol{o}^*] = (\boldsymbol{I}-\boldsymbol{A}_n\boldsymbol{W})^{-1}(\boldsymbol{I}-\boldsymbol{A}_n)(\boldsymbol{F}_n+\boldsymbol{\Gamma})^{-1}\Big(\boldsymbol{\Gamma}\boldsymbol{s}^{\langle 0\rangle}+\mu^*\boldsymbol{f}_n\Big), \tag{13}$$

$$\mathbb{E}\left[p_{\boldsymbol{x}=1}^*\right] = \frac{1}{n}\sum_{i=1}^{n}P\left(x_i^{\langle t\rangle}=1|o_i^{\langle t\rangle}=o_i^*\right), \tag{14}$$

$$\mathbb{E}\left[\boldsymbol{z}^*\right] = \boldsymbol{\omega}\big(\mathbb{E}\big[p_{\boldsymbol{x}=\boldsymbol{1}}^*\big]\big), \tag{15}$$

$$\mathbb{E}\left[\boldsymbol{d}^*\right] = \mathbb{E}\left[\boldsymbol{z}^*\right], \tag{16}$$

where all derivations are in the Supplementary [31]. $\mu^* = \frac{1}{m}\sum_{j=1}^{m}\mathbb{E}[d_j^*]$ is the mean of steady states for dominant agents' actions, $\boldsymbol{f}_n = (F_{n1}, \cdots, F_{nn})^T$ is an $n \times 1$ column vector whose elements are the diagonal elements of $\boldsymbol{F}_n$, $P\left(x_i^{\langle t\rangle}=1|o_i^{\langle t\rangle}=o_i^*\right)$ is the probability of adopting the action 1 when the opinion is asymptotically stable, and its detailed representation is in the Supplementary [31], and $\omega_i\big(\mathbb{E}\big[p_{\boldsymbol{x}=\boldsymbol{1}}^*\big]\big) = \left[1+\left(\frac{1}{\mathbb{E}\left[p_{\boldsymbol{x}=1}^*\right]}-1\right)^{\epsilon_i}\right]^{-1}$. To simplify the analysis in this work, we consider the simple scenario where the trade-off coefficients of different normal agents are equal, i.e.,

$\lambda_1 = \cdots = \lambda_n = \lambda$. In this simple scenario, we can see that the expectation of dominant agents' actions are the same as that of opinions, and the dominant agents' opinions are the result of the distortion for the proportion of normal agents' action. We will investigate the more complicated scenario where $\lambda_i \neq \lambda_j, \exists i \neq j$ in our future work.

Due to the large number of equations and the complexity of (13)-(16), we cannot obtain the theoretical solutions. We propose Algorithm 1 to numerically find the steady states (SS).

Algorithm 1: SS-learning Algorithm

Input: W, A_n, F_n, Γ, learning and decay rate r_l, r_d, error threshold e_t

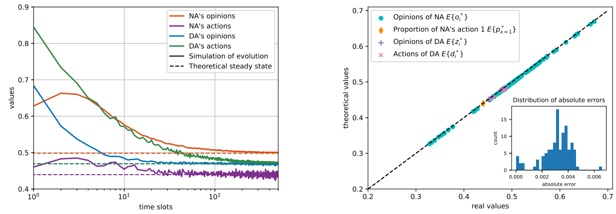
Output: the expectation of steady states for normal agents' and dominant agents' opinions and actions $\mathbb{E}[\boldsymbol{o}^*], \mathbb{E}\left[p^*_{\boldsymbol{x}=1}\right], \mathbb{E}[\boldsymbol{z}^*], \mathbb{E}[\boldsymbol{d}^*]$

Set initial error $e = +\infty$, initial predicted proportion of normal agents' action $p^0_{\boldsymbol{x}=1} = 0.5$, initial gradient direction $D = 0$ and iteration times $It = 0$;
while $e > e_t$ **do**
    $It = It + 1$;
    Update current predicted proportion of normal agents' action by $p^{It}_{\boldsymbol{x}=1} = p^{It-1}_{\boldsymbol{x}=1} + D * r_l * (r_d)^{It}$;
    while $p^{It}_{\boldsymbol{x}=1} \notin [0,1]$ **do**
        $It = It + 1$, $p^{It}_{\boldsymbol{x}=1} = p^{It-1}_{\boldsymbol{x}=1} + D * r_l * (r_d)^{It}$
    Use the current predicted proportion $p^{It}_{\boldsymbol{x}=1}$ to compute $\mathbb{E}[\boldsymbol{z}^*]$ using (15);
    Use the current predicted $\mathbb{E}[\boldsymbol{d}^*]$, i.e. $\mathbb{E}[\boldsymbol{z}^*]$, to compute $\mathbb{E}[\boldsymbol{o}^*]$ using (13);
    Use the current predicted $\mathbb{E}[\boldsymbol{o}^*]$ to compute $P\left(x_i^{\langle t\rangle} = 1 | o_i^{\langle t\rangle} = o_i^*\right)$ using simplified formulas in Supplementary [31];
    Compute the error $e = \frac{1}{n}\sum_{i=1}^{n} P\left(x_i^{\langle t\rangle} = 1 | o_i^{\langle t\rangle} = o_i^*\right) - p^{It}_{\boldsymbol{x}=1}$;
    $D = \text{sgn}(e)$, where $\text{sgn}(\cdot)$ is the sign function;
return $p^{It}_{\boldsymbol{x}=1}$ as $\mathbb{E}\left[p^*_{\boldsymbol{x}=1}\right]$ and compute other steady states by (13), (15) and (16)

IV. Simulations Results

In this section, we run simulations to verify our theoretical analysis. We assume that there are $n = 100$ normal agents and $m = 5$ dominant agents, and normal agents are in an Erdős–Rényi (ER) random weighted network with connecting probability $p = 0.4$. Normal agents' initial opinions $\boldsymbol{s}^{\langle 0\rangle}$ are set randomly in $[0, 1]$. Diagonal elements of the influence matrices and other coefficient matrices, including $\boldsymbol{F}_n$, $\boldsymbol{F}_d$, $\boldsymbol{A}_n$ and $\boldsymbol{\Gamma}$, take random values in $[0, 1]$.The expansion coefficient is $\kappa = 6$. Dominant agents' distortion of the proportion of the action ϵ_i is independently and uniformly distributed in $[0.2, 1]$, which reflects dominant agents' subjective cognitive ability. In each simulation run, the update process is repeated until opinions and actions reach the stable state.

We first observe the whole co-evolution process and validate our theoretical analysis. Fig. 2 shows the simulation and



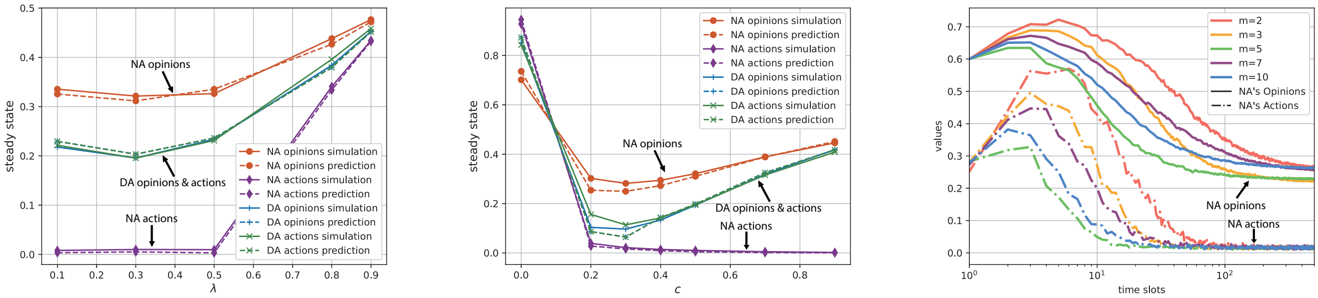
(a) Evolution process and theoretical steady state of normal and dominant agents' opinions and actions.

(b) The comparison of real values and theoretical values and the distribution of all absolute errors.

Fig. 2: Simulation and theoretical results with $\lambda = 0.9$, $c = 0.5$. NA represents normal agents. DA represents dominant agents.

theoretical results when $\lambda = 0.9$ and $c = 0.5$. In Fig. 2(a), solid lines are the mean results with 200 repeated simulations, and dotted lines are the predicted theoretical steady states. We can see that as time goes, simulation results align well with the theoretical steady states. Note that dominant agents' opinions and actions converge to the same point, consistent with (16). Also, the steady states of normal agents' opinions and actions are inconsistent, validating the value-action gap. We can also see that the rate of convergence of dominant agents' opinions is quicker than others. In Fig. 2(b), we scatter plot pairs of real steady states and theoretical analysis results derived by (13)-(16) for each normal agent's and dominant agent's opinions and actions. They are all on or very close to the dashed black line $y = x$, validating our theoretical analysis again.

We then show the impact of different parameters in our model on the final steady state and evolution process. Fig. 3(a) shows the influence of λ on the steady states, which is the trade-off coefficients between normal agent's opinion and the cost, in 5 situations: $\lambda = 0.1, 0.3, 0.5, 0.8, 0.9$. As expected, the steady states of all agents' opinions and actions increase with λ since the cost is more unimportant. The simulation results and predicted steady states match well, which validates our analysis again. Fig. 3(b) shows the effect of c to steady states, which is the additional loss for the action 1, in 7 situations: $c = 0, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9$. We can see that when c increases, fewer normal agents adopt the action 1 due to the rise of additional loss, while other variables including normal agents' opinions, dominant agents' opinions and actions decrease first and then increase. This indicates that the additional loss aggravates normal agent's value-action gap. To avoid the deep gap, dominant agents should take actions to diminish the loss of execution. Fig. 3(c) shows the effect of m to the evolution process, or equivalently m/n, which is the proportion of dominant agents to normal agents, in 5 situations: $m = 2, 3, 5, 7, 10$ with $n = 100$. The results of normal agents' opinions and actions are shown as the example. It can be found that the speed of evolution reaching the final steady states does not monotonically increase with the number of agents. In these 5 situations, normal agents achieve the optimal results from the perspective of dominant agents when $m = 5$, as the co-evolution being stable more quickly. This suggests that dominant agents should arrange their resources and scale in a reasonable range.



(a) The effect of different trade-off coefficients between normal agent's opinion and the cost $\lambda = 0.1, 0.3, 0.5, 0.8, 0.9$.

(b) The effect of different additional loss for the action 1 $c = 0.0, 0.2, 0.3, 0.4, 0.5, 0.7, 0.9$.

(c) The effect of different numbers of dominant agents $m = 2, 3, 5, 7, 10$, i.e., the effect of the proportion of dominant agents to normal agents.

Fig. 3: The simulations and theoretical results of steady states and evolution process in different settings.

V. Conclusions

In this paper, we propose a unified co-evolution model of opinions and actions among dominant and normal agents to study how heterogeneous agents' opinions and actions interact. We incorporate bounded rationality and the value-action gap into our framework. We theoretically analyze the dynamics and steady state of both normal and dominant agents' opinions and actions. Simulations are conducted to validate our analysis, and show the effect of model parameters including λ, c and m/n, indicating that dominant agents should take actions to avoid deeper value-action gap and also arrange their resources properly to accelerate the process to the steady state, providing references for the practical decision.

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