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New Perspectives and Insights on Distortionless Microphone Array Beamforming

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***Abstract*—This paper studies distortionless beamforming within a general signal model, where a target source is accompanied by multiple interference sources and ambient noise. In this context, the minimum variance distortionless response (MVDR) and linearly constrained minimum variance (LCMV) beamformers are commonly used techniques. However, the MVDR beamformer lacks control over residual interference sources, while the LCMV beamformer may result in insufficient noise attenuation and, in some cases, even amplify ambient noise at its output. To provide a flexible mechanism for balancing ambient noise reduction and interference suppression, we introduce a novel optimization criterion, which leads to the development of a general distortionless beamformer that includes both the MVDR and LCMV methods as special cases. The effectiveness of our proposed approach is demonstrated through various performance metrics and beampatterns.**

I. Introduction

In real-world environments, the signal recorded by a microphone often consists of the desired signal from the target source, along with one or more interference signals and pervasive ambient noise. To extract the desired signal and effectively eliminate the unwanted components, noise reduction techniques are commonly used. Among these techniques, beamforming with a microphone array, a method of spatial filtering, is frequently used due to its ability to reduce noise while minimizing distortion of the desired signal [1, 2].

Beamforming methods can be broadly classified based on whether they distort the desired signal. In the category where distortion of the desired signal occurs, representative algorithms include the Wiener beamformer (also known as the multichannel Wiener filter) [3], along with its parameterized variants [4–7], among others. This paper focuses on the category of distortionless beamforming, which aims to preserve the desired signal while suppressing unwanted components. Key algorithms in this category include fixed beamformers, such as the widely used delay-and-sum [3] and superdirective [8] beamformers, as well as adaptive methods like the minimum variance distortionless response (MVDR) beamformer [9] and the linearly constrained minimum variance (LCMV) beamformer [10]. The MVDR beamformer is designed to minimize the variance of the output interference-plus-noise while maintaining a distortionless constraint [11]. However, it often leaves some residual interference, especially in high ambient noise conditions. On the other hand, the LCMV beamformer aims to completely eliminate the effect of interference sources by imposing additional linear constraints and minimizing the residual noise variance [11–15]. Nonetheless, this approach may lead to less effective noise attenuation and, in some cases, amplify the ambient noise due to a reduced degree of freedom [16].

Several methods have been developed in the literature to balance background noise reduction and interference attenuation [16–19]. However, many of these methods focus primarily on scenarios involving a single interference source. For instance, the work in [16] introduced a parameterized distortionless beamformer, which is expressed as a weighted sum of the LCMV beamformer and a matched beamformer (an MVDR solution in the absence of interference). This approach allows for a tradeoff between the two functions by adjusting the weighting coefficient. In [17], an LCMV-like beamformer with specific constraints was developed to balance target signal distortion, noise reduction, and interference suppression. More recently, [19] presented a flexible beamformer that adjusts an "interference attenuation factor" to achieve a balance between interference suppression and ambient noise reduction. For scenarios involving multiple interference sources, the work [18] proposed a controllable LCMV beamformer with a variable number of constraints. This method separates interference sources into two groups: one group is completely nulled out while the other is suppressed along with the ambient noise. While this approach partially balances noise reduction and interference rejection, it does not offer full flexibility for individually controlling each interference source.

In this paper, we consider a general signal model where a desired signal source is present alongside multiple interference sources and ambient noise. We explore the problem of distortionless beamforming within this model and define key performance metrics clearly. Specifically, we introduce a novel optimization criterion featuring multiple design parameters, with each parameter associated with a particular interference source. Using this criterion, we develop a general distortionless beamformer that includes both the MVDR and LCMV beamformers as special cases. By adjusting the design parameters individually, our approach allows for varying levels of tradeoff between ambient noise reduction and interference suppression. The effectiveness and flexibility of this approach are demonstrated through simulations involving one target source and two interference sources.

II. Signal Model and Problem Formulation

Consider a two-dimensional (2-D) sensor array consisting of $M \geq 3$ omnidirectional microphones arranged on a 2-D plane, which may or may not form a regular geometric pattern[1].

[1]Extending the method developed in this paper to three-dimensional arrays is straightforward.

The positions of all microphones are assumed to be known precisely, and all sources (both desired and interference) lie on the same plane as the 2-D array. Angles on this plane, measured from the horizontal axis to a point, are referred to as azimuth angles (θ). We assume there is a single desired signal arriving at the array from the direction θ_0, while there are N interference sources coming from directions θ_n, where $n = 1, 2, \ldots, N$ and $\theta_0 \neq \theta_1 \neq \cdots \neq \theta_N$. Consequently, the frequency-domain observed signal vector of length M at temporal frequency f is given by [2]

$$\begin{aligned} \mathbf{y}(f) &= \begin{bmatrix} Y_1(f) & Y_2(f) & \cdots & Y_M(f) \end{bmatrix}^T \\ &= \mathbf{d}_{\theta_0}(f)X_0(f) + \sum_{n=1}^{N} \mathbf{d}_{\theta_n}(f)X_n(f) + \mathbf{v}(f), \end{aligned} \quad (1)$$

where $Y_m(f)$ is the mth ($m = 1, 2, \ldots, M$) microphone signal of the 2-D array, $\mathbf{d}_{\theta_0}(f)$ is the signal propagation vector at θ_0 (direction of the desired source), $X_0(f)$ is the (zero-mean) desired signal, $\mathbf{d}_{\theta_n}(f)$ is the signal propagation vector at θ_n (direction of the nth interference source), $X_n(f)$ is the (zero-mean) nth interference signal, $\mathbf{v}(f)$ is the (zero-mean) background noise signal vector defined similarly to $\mathbf{y}(f)$, and the superscript ^{T} is the transpose operator. We assume that all sources are mutually independent, and incoherent with $\mathbf{v}(f)$. In the rest, in order to simplify the notation, we drop the dependence on the frequency, f. We deduce that the covariance matrix of $\mathbf{y}$ is

$$\begin{aligned} \mathbf{\Phi}_{\mathbf{y}} &= E\left(\mathbf{y}\mathbf{y}^H\right) \\ &= \phi_{X_0}\mathbf{d}_{\theta_0}\mathbf{d}_{\theta_0}^H + \sum_{n=1}^{N} \phi_{X_n}\mathbf{d}_{\theta_n}\mathbf{d}_{\theta_n}^H + \mathbf{\Phi}_{\mathbf{v}}, \end{aligned} \quad (2)$$

where $E(\cdot)$ denotes mathematical expectation, the superscript ^{H} is the conjugate-transpose operator, $\phi_{X_0} = E\left(|X_0|^2\right)$ is the variance of X_0, $\phi_{X_n} = E\left(|X_n|^2\right)$ is the variance of X_n, and $\mathbf{\Phi}_{\mathbf{v}} = E\left(\mathbf{v}\mathbf{v}^H\right)$ is the covariance matrix of $\mathbf{v}$. For a small and compact 2-D array, we can assume that the noise variance is uniform across all sensors, i.e., $\phi_V = \phi_{V_1} = \phi_{V_2} = \cdots = \phi_{V_M}$, with $\phi_{V_m} = E\left(|V_m|^2\right)$, $m = 1, 2, \ldots, M$; in this case, we can express (2) as

$$\begin{aligned} \mathbf{\Phi}_{\mathbf{y}} &= \phi_{X_0}\mathbf{d}_{\theta_0}\mathbf{d}_{\theta_0}^H + \sum_{n=1}^{N} \phi_{X_n}\mathbf{d}_{\theta_n}\mathbf{d}_{\theta_n}^H + \phi_V\mathbf{\Gamma}_{\mathbf{v}} \\ &= \phi_{X_0}\mathbf{d}_{\theta_0}\mathbf{d}_{\theta_0}^H + \mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H + \phi_V\mathbf{\Gamma}_{\mathbf{v}}, \end{aligned} \quad (3)$$

where $\mathbf{\Gamma}_{\mathbf{v}} = \mathbf{\Phi}_{\mathbf{v}}/\phi_V$ is the coherence matrix of the noise,

$$\mathbf{D} = \begin{bmatrix} \mathbf{d}_{\theta_1} & \mathbf{d}_{\theta_2} & \cdots & \mathbf{d}_{\theta_N} \end{bmatrix}$$

is a rectangular matrix of size $M \times N$, and

$$\boldsymbol{\Phi} = \mathrm{diag}\left(\phi_{X_1}, \phi_{X_2}, \ldots, \phi_{X_N}\right)$$

is a diagonal matrix of size $N \times N$. At this point, we do not specify the relationship between N and M; this will be discussed later.

From (3), we can determine the following measures: the input signal-to-noise ratio (SNR), input signal-to-interference-plus-noise ratio (SINR), input signal-to-interference ratio (SIR), and input noise-to-interference ratio (NIR). They are given by

$$\mathrm{iSNR} = \frac{\phi_{X_0}}{\phi_V}, \quad (4)$$

$$\mathrm{iSINR} = \frac{\phi_{X_0}}{\sum_{n=1}^{N} \phi_{X_n} + \phi_V}, \quad (5)$$

$$\mathrm{iSIR} = \frac{\phi_{X_0}}{\sum_{n=1}^{N} \phi_{X_n}}, \quad (6)$$

$$\mathrm{iNIR} = \frac{\phi_V}{\sum_{n=1}^{N} \phi_{X_n}}. \quad (7)$$

As a result, we have

$$\mathrm{iSINR} = \frac{\mathrm{iSIR}}{1 + \mathrm{iNIR}} = \frac{\mathrm{iSNR}}{1 + \mathrm{iNIR}^{-1}}$$

and

$$\mathrm{iSIR} = \mathrm{iNIR} \times \mathrm{iSNR}.$$

The objective of this paper is to give another perspective on distortionless beamforming based on the general signal model given in (1).

III. Beamforming and Performance Measures

Let $\mathbf{h}$ be a complex-valued linear filter of length M. The most effective method for performing beamforming involves applying $\mathbf{h}$ to the observed signal vector, $\mathbf{y}$, i.e.,

$$\begin{aligned} Z &= \mathbf{h}^H\mathbf{y} \\ &= X_{\mathrm{fd}} + X_{\mathrm{ri}} + V_{\mathrm{rn}}, \end{aligned} \quad (8)$$

where Z is the beamformer output, also an estimate of X_0, $X_{\mathrm{fd}} = X_0\mathbf{h}^H\mathbf{d}_{\theta_0}$ is the filtered desired signal, $X_{\mathrm{ri}} = \sum_{n=1}^{N} X_n\mathbf{h}^H\mathbf{d}_{\theta_n}$ is the residual interference, and $V_{\mathrm{rn}} = \mathbf{h}^H\mathbf{v}$ is the residual noise. Then, the variance of Z is

$$\begin{aligned} \phi_Z &= \mathbf{h}^H\mathbf{\Phi}_{\mathbf{y}}\mathbf{h} \\ &= \phi_{X_{\mathrm{fd}}} + \phi_{X_{\mathrm{ri}}} + \phi_{V_{\mathrm{rn}}}, \end{aligned} \quad (9)$$

where $\phi_{X_{\mathrm{fd}}} = \phi_{X_0}\left|\mathbf{h}^H\mathbf{d}_{\theta_0}\right|^2$ is the variance of the filtered desired signal, $\phi_{X_{\mathrm{ri}}} = \sum_{n=1}^{N} \phi_{X_n}\left|\mathbf{h}^H\mathbf{d}_{\theta_n}\right|^2 = \mathbf{h}^H\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\mathbf{h}$ is the variance of the residual interference, and $\phi_{V_{\mathrm{rn}}} = \mathbf{h}^H\mathbf{\Phi}_{\mathbf{v}}\mathbf{h}$ is the variance of the residual noise. Also, from (8), we clearly see that the distortionless constraint is

$$\mathbf{h}^H\mathbf{d}_{\theta_0} = 1. \quad (10)$$

We emphasize that this study focuses solely on distortionless beamforming; therefore, (10) must always be satisfied. Now, from (9), we find that the output SNR is

$$\mathrm{oSNR}(\mathbf{h}) = \frac{\phi_{X_{\mathrm{fd}}}}{\phi_{V_{\mathrm{rn}}}} = \frac{\phi_{X_0}\left|\mathbf{h}^H\mathbf{d}_{\theta_0}\right|^2}{\mathbf{h}^H\mathbf{\Phi}_{\mathbf{v}}\mathbf{h}}. \quad (11)$$

In the same way, the output SINR is

$$\mathrm{oSINR}(\mathbf{h}) = \frac{\phi_{X_{\mathrm{fd}}}}{\phi_{X_{\mathrm{ri}}} + \phi_{V_{\mathrm{rn}}}} = \frac{\phi_{X_0}\left|\mathbf{h}^H\mathbf{d}_{\theta_0}\right|^2}{\mathbf{h}^H\mathbf{D}\boldsymbol{\Phi}\mathbf{D}^H\mathbf{h} + \mathbf{h}^H\mathbf{\Phi}_{\mathbf{v}}\mathbf{h}}. \quad (12)$$

We also deduce that the output SIR and output NIR are, respectively,

$$\text{oSIR}(\mathbf{h}) = \frac{\phi_{X_0} \left|\mathbf{h}^H \mathbf{d}_{\theta_0}\right|^2}{\mathbf{h}^H \mathbf{D} \mathbf{\Phi} \mathbf{D}^H \mathbf{h}} \tag{13}$$

and

$$\text{oNIR}(\mathbf{h}) = \frac{\mathbf{h}^H \mathbf{\Phi}_{\mathbf{v}} \mathbf{h}}{\mathbf{h}^H \mathbf{D} \mathbf{\Phi} \mathbf{D}^H \mathbf{h}}. \tag{14}$$

We also have the following relationships:

$$\text{oSINR}(\mathbf{h}) = \frac{\text{oSIR}(\mathbf{h})}{1 + \text{oNIR}(\mathbf{h})} = \frac{\text{oSNR}(\mathbf{h})}{1 + \text{oNIR}^{-1}(\mathbf{h})}$$

and

$$\text{oSIR}(\mathbf{h}) = \text{oNIR}(\mathbf{h}) \times \text{oSNR}(\mathbf{h}).$$

Ideally, we aim to determine $\mathbf{h}$ such that $\text{oSNR}(\mathbf{h}) \geq \text{iSNR}$ and $\text{oSIR}(\mathbf{h}) \geq \text{iSIR}$, while achieving an optimal balance between output SNR and output SIR.

IV. Proposed Criterion and Beamformer

Let

$$\mathbf{Q} = \text{diag}(Q_1, Q_2, \ldots, Q_N) \tag{15}$$

be a diagonal matrix of size $N \times N$, where $Q_n \geq 0,\ n = 1, 2, \ldots, N$. The proposed method for general distortionless beamforming is reformulated as the following constrained optimization problem:

$$\min_{\mathbf{h}} \mathbf{h}^H \left(\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H\right) \mathbf{h} \quad \text{s. t.} \quad \mathbf{h}^H \mathbf{d}_{\theta_0} = 1. \tag{16}$$

The solution to the above optimization problem yields the following optimal distortionless beamformer:

$$\mathbf{h}_{\text{o}} = \frac{\left(\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H\right)^{-1} \mathbf{d}_{\theta_0}}{\mathbf{d}_{\theta_0}^H \left(\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H\right)^{-1} \mathbf{d}_{\theta_0}}. \tag{17}$$

Using the Sherman-Morrison formula, the inverse of $\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H$ can be decomposed as

$$\begin{aligned}
&\left(\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H\right)^{-1} \\
&= \mathbf{\Phi}_{\mathbf{v}}^{-1} - \mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\mathbf{Q}\left(\mathbf{I}_N + \mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\mathbf{Q}\right)^{-1}\mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1} \\
&= \mathbf{\Phi}_{\mathbf{v}}^{-1} - \mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\left(\mathbf{Q}^{-1} + \mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\right)^{-1}\mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1} \\
&= \mathbf{\Phi}_{\mathbf{v}}^{-1} - \mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\left(\mathbf{\Sigma} + \mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1}\mathbf{D}\mathbf{\Phi}\right)^{-1}\mathbf{D}^H\mathbf{\Phi}_{\mathbf{v}}^{-1},
\end{aligned} \tag{18}$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix and

$$\mathbf{\Sigma} = \text{diag}(\varsigma_1,\ \varsigma_2,\ \ldots,\ \varsigma_N)$$

is a diagonal matrix of size $N \times N$ with $\varsigma_n = 1/Q_n$, $0 \leq \varsigma_n \leq 1$, and $n = 1, 2, \ldots, N$.

The decomposition in (18) is more intuitive because it reveals several important special cases. Indeed, for $\mathbf{\Sigma} = \mathbf{I}_N$ $(\forall M, N)$, we have $\mathbf{h}_{\text{o}} = \mathbf{h}_{\text{MVDR}}$, which is the MVDR beamformer [3], and for $\mathbf{\Sigma} = \text{diag}(0,\ 0,\ \ldots,\ 0)$, with $M > N$, we have $\mathbf{h}_{\text{o}} = \mathbf{h}_{\text{LCMV}}$, which is the LCMV beamformer [3]; indeed, one can verify in this case that $\mathbf{D}^H\left(\mathbf{\Phi}_{\mathbf{v}} + \mathbf{D}\mathbf{\Phi}\mathbf{Q}\mathbf{D}^H\right)^{-1} = \mathbf{0}$.

By individually adjusting the values of the ς_n's, we can achieve various trade-offs between background noise attenuation and interference suppression. This new beamformer offers significantly greater flexibility compared to the conventional MVDR and LCMV beamformers, as it encompasses both methods.

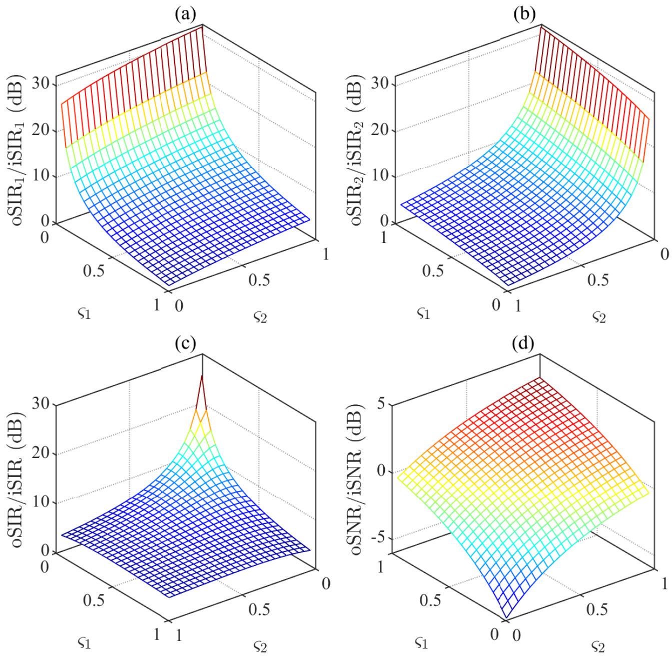


Fig. 1. Performance of the proposed beamformer as a function of ς_1 and ς_2: (a) gain in SIR_1, (b) gain in SIR_2, (c) gain in SIR, and (d) gain in SNR. Conditions: $f = 1$ kHz, $\Delta_\theta = 30°$, and $\text{iSIR}_1 = \text{iSIR}_2 = \text{iSNR} = 10$ dB.

V. Simulations

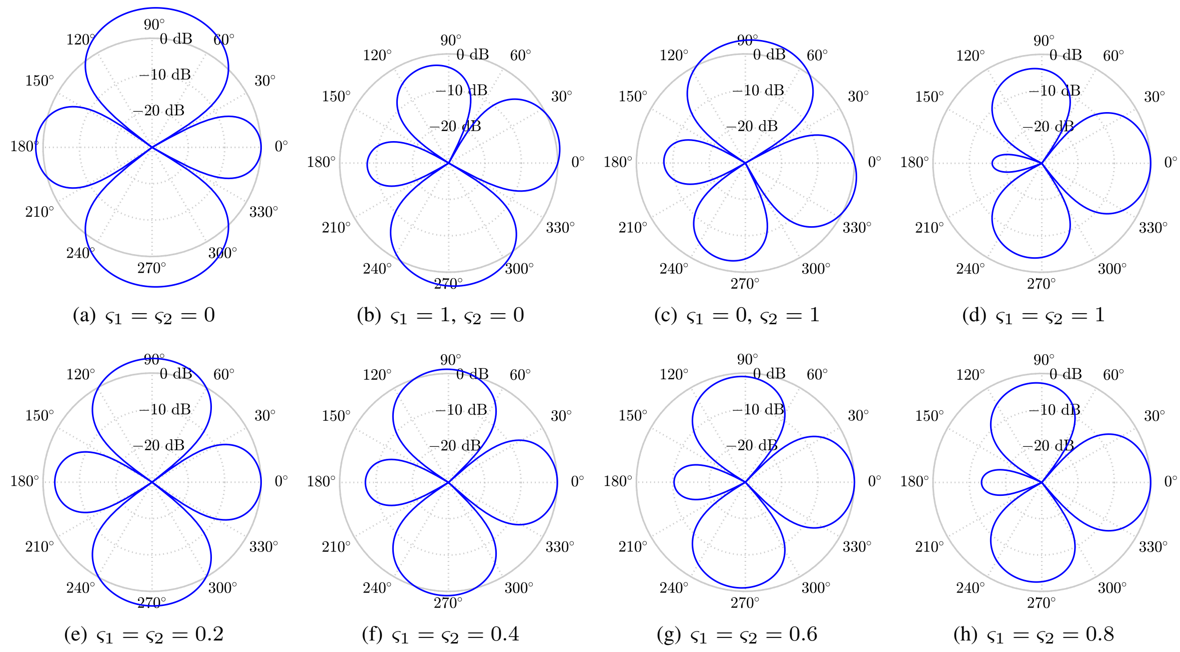
In this section, we evaluate the performance of the developed distortionless beamformer through numerical simulations using various performance metrics. We examine the trade-offs between background noise reduction and interference suppression, and analyze the beampattern behavior, as illustrated in previous studies [3, 16, 20].

We consider a uniform circular array (UCA) with $M = 8$ microphone sensors positioned in an anechoic environment. The UCA has a radius of 4 cm. There are one desired source and two interference sources in the far field of the array. The azimuth angle of the desired source is $\theta_0 = 0°$, while the azimuth angles of the two interference sources are $\theta_1 = \theta_0 + \Delta_\theta$ and $\theta_2 = \theta_0 - \Delta_\theta$, respectively, with $\Delta_\theta > 0$. Taking the center of the UCA as the reference point, the steering vector at angle θ is given by [21]

$$\mathbf{d}_\theta(f) = \left[\begin{array}{ccc} e^{\jmath 2\pi f r/c\cos(\theta - \psi_1)} & \ldots & e^{\jmath 2\pi f r/c\cos(\theta - \psi_M)} \end{array}\right]^T, \tag{19}$$

where $\jmath$ is the imaginary unit with $\jmath^2 = -1$, c is the speed of sound, which is typically 340 m/s, and $\psi_m = 2\pi(m-1)/M$ is the angular position of the mth sensor. The noise field consists of spherically isotropic (diffuse) noise and spatially white noise. In this case, the noise coherence matrix is

$$\mathbf{\Gamma}_{\mathbf{v}}(f) = (1 - \mu)\mathbf{\Gamma}_{\text{d}}(f) + \mu\mathbf{I}_M, \tag{20}$$



(a) $\varsigma_1 = \varsigma_2 = 0$ (b) $\varsigma_1 = 1$, $\varsigma_2 = 0$ (c) $\varsigma_1 = 0$, $\varsigma_2 = 1$ (d) $\varsigma_1 = \varsigma_2 = 1$

(e) $\varsigma_1 = \varsigma_2 = 0.2$ (f) $\varsigma_1 = \varsigma_2 = 0.4$ (g) $\varsigma_1 = \varsigma_2 = 0.6$ (h) $\varsigma_1 = \varsigma_2 = 0.8$

Fig. 2. Beampattern examples of the proposed beamformer for different combinations of ς_1 and ς_2. Conditions: $f = 1$ kHz, $\Delta_\theta = 30°$, and $\text{iSIR}_1 = \text{iSIR}_2 = \text{iSNR} = 10$ dB.

where μ is a real number that relates to the power ratio between the two components of noise, $\mathbf{\Gamma}_\text{d}(f)$ is the coherence matrix for the diffuse noise, and $\mathbf{I}_M$ is the $M \times M$ identity matrix. In our simulations, we set μ to 0.05.

To evaluate the performance of the beamformer on each individual interference source, we calculate the input and output SIRs for the nth interference source as follows:

$$\text{iSIR}_n(f) = \frac{\phi_{X_0}(f)}{\phi_{X_n}(f)} \tag{21}$$

$$\text{oSIR}_n(f) = \frac{\phi_{X_0}(f) \left|\mathbf{h}^H(f)\mathbf{d}_{\theta_0}(f)\right|^2}{\phi_{X_n}(f) \left|\mathbf{h}^H(f)\mathbf{d}_{\theta_n}(f)\right|^2}. \tag{22}$$

Figure 1 plots the performance of the proposed beamformer given in (17) and (18) as a function of the design parameters ς_1 and ς_2. The plot is generated with $\Delta_\theta = 30°$, $f = 1.0$ kHz, and $\text{iSIR}_1 = \text{iSIR}_2 = \text{iSNR} = 10$ dB. Figures 1(a) and (b) show that, for each $n \in \{1, 2\}$, the gain in SIR_n increases as the value of ς_n decreases. When $\varsigma_n = 0$, the beamformer is configured to completely eliminate the nth interference, resulting in a theoretical infinite gain in SIR_n. Figures 1(c) and (d) reveal that decreasing both values of ς_1 and ς simultaneously enhances the overall SIR gain, but this improvement comes at the cost of reduced SNR gain. It is important to note that negative SNR gains may occur with small values of ς_1 and ς, indicating an increase in background noise.

The LCMV and MVDR beamformers represent two specific cases of our proposed beamformer. The LCMV ($\varsigma_1 = \varsigma_2 = 0$) achieves infinite overall SIR gain but results in the lowest SNR gain. In contrast, the MVDR ($\varsigma_1 = \varsigma_2 = 1$) provides the maximum SNR gain but may not deliver sufficient SIR gain. Overall, by adjusting the values of ς_1 and ς_2, our proposed beamformer allows for various trade-offs between different performance gains.

We now analyze the beampattern behavior of the proposed beamformer to explain the observed variations in performance. Figure 2 illustrates the beampatterns for different combinations of ς_1 and ς_2.

Figure 2(a) shows that the LCMV ($\varsigma_1 = \varsigma_2 = 0$) creates perfect nulls at 30° and 330°. However, this results in a pattern issue where some sidelobes have heights significantly greater than unity (the response in the desired source direction), which can account for the amplification of background noise. Figures 2(b) and (c) demonstrate that when $\varsigma_n = 0$, the proposed beamformer achieves a perfect null in the direction of each corresponding interference source, effectively eliminating its impact. Figure 2(d) shows that the MVDR ($\varsigma_1 = \varsigma_2 = 1$) does not create perfect nulls for the interference sources but maintains well-behaved sidelobes with lower levels compared to the main beam. Figures 2(e)–(h) reveal that increasing both ς_1 and ς_2 simultaneously reduces the sidelobe levels but shifts the spatial nulls away from the interference directions.

VI. Conclusions

This paper investigated distortionless beamforming within a general signal model where multiple interference sources and background noise coexist with the desired signal. To offer highly flexible trade-offs between interference rejection and background noise reduction, we proposed a novel optimization criterion and developed a parameterized beamformer that includes the conventional MVDR and LCMV beamformers as special cases. The effectiveness of our proposed beamformer was demonstrated through numerical evaluations using different performance metrics and an analysis of beampattern behavior.

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