Reduced-dimensional MUSIC Algorithm for Frequency Diverse Array in MIMO Radar System

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***Abstract*—The paper proposes a reduced-dimensional multiple signal classification (RD-MUSIC) algorithm for frequency diverse array (FDA) multi-input multi-output (MIMO) radar. The proposed method firstly constructs a constrained minimization problem based on the two-dimensional (2D) MUSIC spectrum function. Then, we reduce the dimension of the minimization problem by calculating the partial derivative with respect to the range parameter, which results in the minimization problem only about the direction of arrival (DOA) parameter. Finally, we substitute the DOA estimates into the steering vector and directly obtain the closed-form solutions of range estimates based on the least squares (LS) method. The proposed algorithm requires only one 1D peak search for DOA parameters, which greatly reduces the computational complexity. In addition, 1D peak searches can be used for the range estimation as well, achieving better estimation accuracy at the cost of the increased complexity compared with the LS solutions. Moreover, the proposed algorithm can be applied to not only uniform linear array (ULA) but also sparse array, demonstrating high versatility. Numerical results show that the proposed RD-MUSIC algorithm is effectiveness regardless of the array configuration and that it can outperform the conventional algorithms including atomic norm minimization (ANM) based method and MUSIC based method.**

I. Introduction

In array signal processing, direction of arrival (DOA) estimation based on the multi-input multi-output (MIMO) radar [1] is widely used in various fields due to the higher spatial degrees of freedom compared to passive antenna arrays. Recently, frequency diverse array (FDA) MIMO radar [2][3] has received attention because of its DOA-range independent beampattern, which enables us to simultaneously obtain DOA and range estimates of multiple targets. Many traditional algorithms have been applied for the joint DOA and range estimation of the FDA MIMO radar so far, such as multiple signal classification (MUSIC) [4], estimation of signal parameters via rotational invariance techniques (ESPRIT) [5], and parallel factor analysis (PARAFAC) [6] based algorithms.

In order to improve the estimation performance, compressed sensing (CS) [7] technique has been applied for the FDA MIMO radar. A representative CS approach called atomic norm minimization (ANM), which is a grid-free algorithm, has been applied to the FDA MIMO radar in [8]. The CS based methods generally can cope with a small number of snapshots, but the algorithm requires prohibitive computational complexity, since the estimation problem is formulated as a semi-definite programming (SDP) problem [9]. To reduce the complexity while keeping the estimation performance, the idea of sparse array has been utilized in [10], but it still requires high computational complexity. Moreover, the algorithm requires the transmitting array to be a uniform linear aray (ULA) in order to ensure that the Vandermonde decomposition of the correlation matrix is possible.

Some improved algorithms based on the traditional approaches also has been proposed for the FDA MIMO radar. An improved ESPRIT based algorithm is proposed in [11], where DOA estimates are used to recover the range steering vector from the estimated receive-transmit array manifold. The method requires slightly higher complexity than the classical ESPRIT algorithm, but achieves better range estimation performance when nonlinear frequency increments are applied for the FDA. One of restrictions of the algorithm is that it requires the array structure to be the ULA. On the other hand, a MUSIC based algorithm for the FDA MIMO radar is proposed in [12], called reduced-dimensional (RD) MUSIC algorithm, where a two-dimensional (2D) peak search the MUSIC spectrum function for the DOA and range estimation is converted into several one-dimensional (1D) peak searches by the proposed constrained optimization problem, which can reduce the computational complexity of the original 2D MUSIC algorithm. The algorithm does not require the ULA, and can be applied for sparse arrays to improve the performance. However, the required number of the 1D peak searches for the range estimates is the same as the number of targets, whose computational complexity might not be acceptable in some applications.

In this paper, to further reduce the computational complexity of the RD MUSIC in [12], an improved MUSIC based algorithm for the FDA MIMO radar is proposed. Different from [12], the proposed method uses the ℓ_2-norm of the transmitting steering vector as a constraint of the optimization problem, which enables us to fully utilize all information in the received signal in the DOA peack search process. Moreover, for the computational complexity reduction, the range estimates are

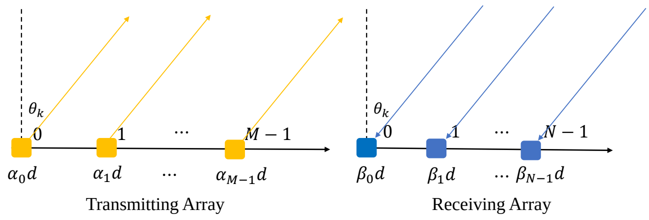

Fig. 1. An example of FDA for co-located MIMO radar

given by closed-form solutions based on the LS method. Additionally, 1D peak searches can be also applied to obtain the range estimates with higher accuracy at the cost of increased complexity compared with the closed-form LS solutions. The performance of the proposed algorithm for both ULA and sparse arrays is verified by computer simulations comparing with that of conventional methods.

In the rest of paper, $\mathbb{N}$ and $\mathbb{C}$ denote sets of all natural numbers and complex numbers, respectively. We use lower-case bold characters to denote column vectors and upper-case bold characters for matrices. $(\cdot)^T$ and $(\cdot)^H$ are the transpose and conjugate transpose of a matrix or vector, respectively. $(\cdot)^{-1}$ denotes the inverse of a matrix. $\oplus$, $\otimes$, and $\odot$ represent Hadamard product, Kronecker product, and Khatri-Rao product, respectively. $\|\cdot\|_2$ denotes the ℓ_2-norm and $|\cdot|$ is the absolute value. $\ln(\cdot)$ denotes the natural logarithm.

II. System Model of FDA-MIMO Radar

Fig. 1 illustrates the co-located FDA-MIMO radar considered in this paper. The transmitting array is a linear array consisting of M antenna elements. Suppose that the 0-th transmitting antenna is located at the origin, and the location set of the transmitting antenna elements is defined as

$$\Phi_t = \{\alpha_0 d, \alpha_1 d, \ldots, \alpha_{M-1} d\}, \tag{1}$$

where $\alpha_0 = 0$, $\alpha_m \in \mathbb{N}$ $(m = 1, \ldots, M-1)$, and d denotes unit spacing between antenna elements.

A linear frequency shift corresponding to the antenna location is employed for the FDA, and the transmitting frequency of the m-th antenna is given by

$$f_m = f_0 + \alpha_m \Delta f, \; (m = 0, 1, \ldots, M-1), \tag{2}$$

where f_0 denotes the carrier frequency at the 0-th transmitting antenna and $\Delta f (\ll f_0)$ is the frequency shift corresponding to the unit antenna spacing d. To avoid phase ambiguity in DOA estimation, d is set to be the half of the smallest wavelength among the transmitted signals as

$$d = \frac{c}{2(f_0 + \alpha_{M-1}\Delta f)}, \tag{3}$$

where c is the speed of light. Note that, for the case with a sparse array, at least one antenna spacing must be d as a limitation.

Based on the assumptions above, the transmitted signal from the m-th transmitting antenna can be written as

$$x_m(t) = \phi_m(t) e^{j2\pi(f_0 + \alpha_m \Delta f)t}, \; (0 \le t \le T), \tag{4}$$

where T is the duration and $\phi_m(t)$ is the orthogonal baseband waveform from the m-th transmitting antenna, which has properties of

$$\int_0^T \phi_{m_1}(t)\phi_{m_2}^*(t-\tau)dt = \begin{cases} 0 & (m_1 \neq m_2) \\ 0 & (m_1 = m_2, \tau \neq 0) \\ 1 & (m_1 = m_2, \tau = 0) \end{cases}. \tag{5}$$

The receiving array depicted in Fig. 1 having N antenna elements is also assumed to be a linear array, and the location set of the receiving antenna elements is expressed as

$$\Phi_r = \{\beta_0 d, \beta_1 d, \ldots, \beta_{N-1} d\}, \tag{6}$$

where β_0 is assumed as 0 and $\beta_n \in \mathbb{N}$ $(n = 1, \ldots, N-1)$.

Suppose that there exist K far field targets in the space with DOA and range parameters of (θ_k, r_k) , $(k = 0, 1, \cdots, K-1)$ with respect to the antenna origin. All the targets are illuminated by the signals transmitted from the FDA, and reflect them back to the receiving array. During the process, taking the 0-th transmitting and receiving elements as the references, for a specific k-th target with DOA and range (θ_k, r_k), the time delay from the p-th transmitting antenna to the q-th receiving antenna is given by

$$\tau_{m,n,k} = \tau_{0,0,k} - \left(\frac{\alpha_m d \sin\theta_k}{c} + \frac{\beta_n d \sin\theta_k}{c} \right), \tag{7}$$

where $m = 0, 1, \ldots, M-1$ and $n = 0, 1, \ldots, N-1$. The first term is the reference time delay, the second and third term are resulted from the location of transmitting and receiving antennas, respectively.

Combining (4) and (7) while ignoring the additive noise, the received signal at the n-th receiving antenna corresponding to the k-th target from all the transmitting antennas is given by

$$\bar{y}_{n,k}(t) = \sum_{m=0}^{M-1} \phi_m(t - \tau_{m,n,k}) e^{j2\pi(f_0 + \alpha_m \Delta f)(t - \tau_{m,n,k})}. \tag{8}$$

Assuming transmitted signals to be narrowband, namely $\phi_m(t - \tau_{m,n,k}) = \phi_m(t - \tau_{0,0,k})$, (8) can be rewritten as

$$\begin{aligned} &\bar{y}_{n,k}(t) \\ &= \sum_{m=0}^{M-1} \phi_m(t - \tau_{0,0,k}) e^{j2\pi f_0 t} e^{-j2\pi f_0 \frac{2r_k}{c}} e^{j2\pi f_0 \frac{\alpha_m d \sin\theta_k}{c}} \\ &\quad \cdot e^{j2\pi f_0 \frac{\beta_n d \sin\theta_k}{c}} e^{j2\pi \alpha_m \Delta f t} e^{-j2\pi \alpha_m \Delta f \frac{2r_k}{c}} \\ &\quad \cdot e^{j2\pi \alpha_m \Delta f \frac{\alpha_m d \sin\theta_k}{c}} e^{j2\pi \alpha_m \Delta f \frac{\beta_n d \sin\theta_k}{c}}. \end{aligned} \tag{9}$$

The received signal can be separated by matched filters using the orthogonal properties in (5). Specifically, the received signal at the n-th antenna from the m-th transmitting antenna reflected at the k-th target is obtained by

$$\begin{aligned} & y_{m,n,k} \\ & = \int_0^T \phi_m^* \left(t - \tau_{0,0,k}\right) e^{-j2\pi(f_0+\alpha_m \Delta f)t} \bar{y}_{n,k}(t) dt \\ & = e^{-j2\pi f_0 \frac{2r_k}{c}} e^{j2\pi(f_0+\alpha_m\Delta f)\frac{\alpha_m d \sin\theta_k}{c}} e^{j2\pi(f_0+\alpha_m\Delta f)\frac{\beta_n d\sin\theta_k}{c}} \\ & \quad \cdot e^{-j2\pi\alpha_m \Delta f \frac{2r_k}{c}}. \end{aligned} \tag{10}$$

Since we assume $\Delta f \ll f_0$, we have approximate formulas of

$$\begin{aligned} e^{j2\pi(f_0+\alpha_m\Delta f)\frac{\alpha_m d\sin\theta_k}{c}} &\approx e^{j2\pi f_0 \frac{\alpha_m d \sin\theta_k}{c}}, \\ e^{j2\pi(f_0+\alpha_m\Delta f)\frac{\beta_n d\sin\theta_k}{c}} &\approx e^{j2\pi f_0 \frac{\beta_n d\sin\theta_k}{c}}. \end{aligned}$$

Besides, if we define $\xi_k = e^{-j4\pi f_0 r_k/c}$, the received signal can be expressed as

$$y_{m,n,k} = \xi_k e^{-j4\pi \frac{\alpha_m \Delta f r_k}{c}} e^{j2\pi f_0 \frac{\alpha_m d \sin\theta_k}{c}} e^{j2\pi f_0 \frac{\beta_n d\sin\theta_k}{c}}. \tag{11}$$

Since we have N receiving antennas and M matched filter outputs for each receiving antenna, the received signal corresponding to the k-th target can be written in a matrix form as

$$\mathbf{Y}_k = \begin{bmatrix} y_{0,0,k} & y_{0,1,k} & \cdots & y_{0,N-1,k} \\ y_{1,0,k} & y_{1,1,k} & \cdots & y_{1,N-1,k} \\ \vdots & \vdots & \ddots & \vdots \\ y_{M-1,0,k} & y_{M-1,1,k} & \cdots & y_{M-1,N-1,k} \end{bmatrix}. \tag{12}$$

According to the exponential terms in (11), we can rewrite (12) by a matrix-vector form as

$$\mathbf{Y}_k = \xi_k \left[\boldsymbol{b}_{\mathrm{t}}\left(\theta_k\right) \oplus \boldsymbol{c}_{\mathrm{t}}\left(r_k\right)\right] \boldsymbol{a}_{\mathrm{r}}^T\left(\theta_k\right), \tag{13}$$

where

$$\begin{aligned} \boldsymbol{b}_{\mathrm{t}}(\theta_k) &= \left[e^{j2\pi\alpha_0 d\sin(\theta_k)/\lambda}, \cdots, e^{j2\pi\alpha_{M-1} d\sin(\theta_k)/\lambda}\right]^T, \\ \boldsymbol{c}_{\mathrm{t}}(r_k) &= \left[e^{-j4\pi\alpha_0 \Delta f r_k/c}, \cdots, e^{-j4\pi\alpha_{M-1}\Delta f r_k/c}\right]^T, \\ \boldsymbol{a}_{\mathrm{r}}(\theta_k) &= \left[e^{j2\pi\beta_0 d\sin(\theta_k)/\lambda}, \cdots, e^{j2\pi\beta_{N-1} d\sin(\theta_k)/\lambda}\right]^T, \end{aligned}$$

and $\lambda = c/f_0$.

Taking the impact of the additive Gaussian noise on the received signal into consideration and summing up all the reflected signals from the K targets, the received signal model of the l-th ($l = 0, 1, \cdots, L-1$) snapshot is denoted as

$$\mathbf{Y}(l) = \sum_{k=0}^{K-1} \xi_k(l) \left[\boldsymbol{b}_{\mathrm{t}}\left(\theta_k\right) \oplus \boldsymbol{c}_{\mathrm{t}}\left(r_k\right)\right] \boldsymbol{a}_{\mathrm{r}}^T\left(\theta_k\right) + \mathbf{N}(l), \tag{14}$$

where $\mathbf{N}(l) \in \mathbb{C}^{M\times P}$ is a zero-mean white circular complex Gaussian noise matrix in the l-th snapshot. We give the index l to ξ_k because it takes different value for each snapshot.

By using the property of $\mathrm{vec}(\boldsymbol{a}\boldsymbol{b}^T) = \boldsymbol{b} \otimes \boldsymbol{a}$ where $\mathrm{vec}(\cdot)$ denotes the vectorization of a matrix, (14) can be rewritten by vector form with size $MN \times 1$ as

$$\begin{aligned} \boldsymbol{y}(l) &= \mathrm{vec}(\mathbf{Y}(l)) \\ &= \sum_{k=0}^{K-1} \xi_k(l) \boldsymbol{a}_{\mathrm{r}}\left(\theta_k\right) \otimes \left(\boldsymbol{b}_{\mathrm{t}}\left(\theta_k\right) \oplus \boldsymbol{c}_{\mathrm{t}}\left(r_k\right)\right) + \boldsymbol{n}(l) \\ &= \sum_{k=0}^{K-1} \xi_k(l) \boldsymbol{a}_{\mathrm{r}}\left(\theta_k\right) \otimes \boldsymbol{a}_{\mathrm{t}}\left(\theta_k, r_k\right) + \boldsymbol{n}(l) \\ &= \mathbf{A}_{\mathrm{r}}\left(\boldsymbol{\theta}\right) \odot \mathbf{A}_{\mathrm{t}}\left(\boldsymbol{\theta}, \boldsymbol{r}\right) \boldsymbol{\xi}(l) + \boldsymbol{n}(l), \end{aligned} \tag{15}$$

where

$$\boldsymbol{n}(l) = \mathrm{vec}(\mathbf{N}(l)), \tag{16}$$

$$\boldsymbol{a}_{\mathrm{t}}\left(\theta_k, r_k\right) = \boldsymbol{b}_{\mathrm{t}}\left(\theta_k\right) \oplus \boldsymbol{c}_{\mathrm{t}}\left(r_k\right), \tag{17}$$

$$\mathbf{A}_{\mathrm{t}}\left(\boldsymbol{\theta}, \boldsymbol{r}\right) = \left[\boldsymbol{a}_{\mathrm{t}}\left(\theta_0, r_0\right), \cdots, \boldsymbol{a}_{\mathrm{t}}\left(\theta_{K-1}, r_{K-1}\right)\right], \tag{18}$$

$$\mathbf{A}_{\mathrm{r}}\left(\boldsymbol{\theta}\right) = \left[\boldsymbol{a}_{\mathrm{r}}\left(\theta_0\right), \cdots, \boldsymbol{a}_{\mathrm{r}}\left(\theta_{K-1}\right)\right], \tag{19}$$

$$\boldsymbol{\xi}(l) = \left[\xi_0(l), \cdots, \xi_{K-1}(l)\right]^T, \tag{20}$$

and $\boldsymbol{\theta} = [\theta_0, \cdots, \theta_{K-1}]^T$, $\boldsymbol{r} = [r_0, \cdots, r_{K-1}]^T$. By collecting L snapshots, the final received signal is written as

$$\bar{\mathbf{Y}} = \mathbf{A}_{\mathrm{r}}\left(\boldsymbol{\theta}\right) \odot \mathbf{A}_{\mathrm{t}}\left(\boldsymbol{\theta}, \boldsymbol{r}\right) \boldsymbol{\xi} + \mathbf{N} \in \mathbb{C}^{MN\times L}, \tag{21}$$

where

$$\begin{aligned} \bar{\mathbf{Y}} &= \left[\boldsymbol{y}(0), \cdots, \boldsymbol{y}(L-1)\right], \\ \boldsymbol{\xi} &= \left[\boldsymbol{\xi}(0), \cdots, \boldsymbol{\xi}(L-1)\right], \\ \mathbf{N} &= \left[\boldsymbol{n}(0), \cdots, \boldsymbol{n}(L-1)\right]. \end{aligned}$$

III. Reduced-Dimensional MUSIC Algorithm for DOA and Range Estimation

In this section, an improved RD MUSIC algorithm is proposed for FDA in MIMO radar system introduced in Sect. II. The proposed algorithm can be applied not only to ULA but also sparse arrays. First, the MUSIC spectrum function is reshaped to separate DOA and range parameter. The reformed function is transformed into a constrained minimization problem and the range parameter in the search function is eliminated by using the property of the vanishing derivative. After 1D search for DOA estimates, they are substituted into the minimization problem and the range estimates are obtained by the least squares method. In addition, 1D search can also be supported for range estimates with higher performance.

A. DOA estimation by 1D search

The covariance matrix of the final received signal in (21) can be approximately given by the sample covariance matrix using L snapshots as

$$\mathbf{R} \approx \bar{\mathbf{Y}}\bar{\mathbf{Y}}^H / L. \tag{22}$$

By eigendecomposition, the covariance matrix can be written as

$$\mathbf{R} = \mathbf{U}_{\mathrm{s}} \boldsymbol{\Lambda}_{\mathrm{s}} \mathbf{U}_{\mathrm{s}}^H + \mathbf{U}_{\mathrm{n}} \boldsymbol{\Lambda}_{\mathrm{n}} \mathbf{U}_{\mathrm{n}}^H, \tag{23}$$

where $\mathbf{U}_\mathrm{s} \in \mathbb{C}^{MN\times K}$ and $\mathbf{U}_\mathrm{n} \in \mathbb{C}^{MN\times(MN-K)}$ denote the matrices whose column vectors span signal subspace and noise subspace, respectively, while $\mathbf{\Lambda}_\mathrm{s}$ and $\mathbf{\Lambda}_\mathrm{n}$ are the diagonal matrices with eigenvalues corresponding to the signal subspace and the noise subspace, respectively.

Applying the received signal model in the previous section, the traditional MUSIC spectrum function for peak search is expressed as

$$F(\theta,r)=\frac{1}{\left[\boldsymbol{a}_\mathrm{r}\left(\theta\right)\otimes\boldsymbol{a}_\mathrm{t}\left(\theta,r\right)\right]^H\mathbf{U}_\mathrm{n}\mathbf{U}_\mathrm{n}^H\left[\boldsymbol{a}_\mathrm{r}\left(\theta\right)\otimes\boldsymbol{a}_\mathrm{t}\left(\theta,r\right)\right]}. \tag{24}$$

Since we have $\boldsymbol{a}_\mathrm{r}\left(\theta\right)\otimes\boldsymbol{a}_\mathrm{t}\left(\theta,r\right)=\left[\boldsymbol{a}_\mathrm{r}\otimes\mathbf{I}_M\right]\boldsymbol{a}_\mathrm{t}\left(\theta,r\right)$, where $\mathbf{I}_M$ is an identity matrix of size $M\times M$, the MUSIC spectrum function (24) can be rewritten as

$$F(\theta,r)=\frac{1}{\boldsymbol{a}_\mathrm{t}(\theta,r)^H\mathbf{Q}(\theta)\boldsymbol{a}_\mathrm{t}(\theta,r)}, \tag{25}$$

where

$$\mathbf{Q}(\theta)=\left[\boldsymbol{a}_\mathrm{r}(\theta)\otimes\mathbf{I}_M\right]^H\mathbf{U}_n\mathbf{U}_n^H\left[\boldsymbol{a}_\mathrm{r}(\theta)\otimes\mathbf{I}_M\right] \tag{26}$$

is an $M\times M$ Hermitian matrix depending only on the DOA parameter.

Since the direct 2D search for the MUSIC spectrum function requires prohibitive computational complexity, we transform the MUSIC search process into a minimization problem with constraint as

$$\begin{cases}\min_{\theta,r}\boldsymbol{a}_\mathrm{t}(\theta,r)^H\mathbf{Q}(\theta)\boldsymbol{a}_\mathrm{t}(\theta,r)\\ \text{s.t. } ||\boldsymbol{a}_\mathrm{t}(\theta,r)||_2=M\end{cases}, \tag{27}$$

where the constraint is added to avoid the trivial solution of $\boldsymbol{a}_\mathrm{t}(\theta,r)=0$. The minimization problem with the constraint can be reformulated as the unconstrained minimization problem whose cost function is given by using a Lagrange multiplier μ as

$$L(\theta,r)=\boldsymbol{a}_\mathrm{t}(\theta,r)^H\mathbf{Q}(\theta)\boldsymbol{a}_\mathrm{t}(\theta,r)-\mu\left(||\boldsymbol{a}_\mathrm{t}(\theta,r)||_2-M\right). \tag{28}$$

The partial derivative of $L(\theta,r)$ with respect to r is given by

$$\frac{\partial L(\theta,r)}{\partial r}=\left[\boldsymbol{a}_\mathrm{t}(\theta,r)^H\mathbf{Q}(\theta)-\mu\boldsymbol{a}_\mathrm{t}(\theta,r)^H\right]\mathrm{diag}\left(\boldsymbol{b}_\mathrm{t}(\theta)\right)\cdot\left(-j4\pi\Delta f/c\right)\begin{bmatrix}\alpha_0 e^{-j4\pi\alpha_0\Delta fr/c}\\ \vdots\\ \alpha_{M-1}e^{-j4\pi\alpha_{M-1}\Delta fr/c}\end{bmatrix}. \tag{29}$$

The condition of the vanishing derivative gives

$$\mathbf{Q}(\theta)\boldsymbol{a}_\mathrm{t}(\theta,r)=\mu\boldsymbol{a}_\mathrm{t}(\theta,r), \tag{30}$$

which means that $\boldsymbol{a}_\mathrm{t}(\theta,r)$ is an eigenvector of $\mathbf{Q}(\theta)$ corresponding to the eigenvalue of μ. If we ubstitute (30) into (25), we have

$$F(\theta)=\frac{1}{\mu M}. \tag{31}$$

Thus, we need to choose the smallest eigenvalue $\mu_{\min}(\theta)$ of $\mathbf{Q}(\theta)$ out of M eigenvalues as μ in (31) to maximize the MUSIC spectrum function.

Through the above steps, we have transformed the peak search of the 2D MUSIC spectrum function (24) into the 1D search of (31) with the the smallest eigenvalue $\mu_{\min}(\theta)$, which gives K DOA estimates $\hat{\boldsymbol{\theta}}=[\hat{\theta}_0,\cdots,\hat{\theta}_{K-1}]^T$ and their corresponding eigenvectors $[\overline{\boldsymbol{a}}_\mathrm{t}(\theta_0,r_0),\cdots,\overline{\boldsymbol{a}}_\mathrm{t}(\theta_{K-1},r_{K-1})]$.

B. Range estimation by least squares method

The remaining problem is how to obtain the range estimates from $\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)$ and $\hat{\theta}_k$, $k=0,1,\cdots,K-1$. Basically, we can estimate the ranges by using (17), but the eigenvectors $\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)$ and the actual steering vectors $\boldsymbol{a}_\mathrm{t}\left(\theta_k,r_k\right)$ can be different by a complex constant multiplication factor. In addition, due to the additive noise and/or the estimation error of the covariance matrix, the absolute value of each element in the estimated $\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)$ is not necessarily 1. Thus, in our approach, we propose to use the modified eigenvectors $\hat{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)$, whose m-th element is defined as

$$\hat{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)_m=\frac{\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)_m}{\left|\overline{a}_\mathrm{t}\left(\theta_k,r_k\right)_m\right|\frac{\overline{\boldsymbol{a}}_\mathrm{t}(\theta_k,r_k)_0}{|\overline{a}_\mathrm{t}(\theta_k,r_k)_0|}}, \tag{32}$$

where $\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)_m$ denotes the m-th element of $\overline{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)$.

According to (17), $\boldsymbol{c}_\mathrm{t}\left(r_k\right)$, which contains the information of the range r_k in the power of each element, can be given by

$$\boldsymbol{c}_\mathrm{t}\left(r_k\right)=\boldsymbol{a}_\mathrm{t}\left(\theta_k,r_k\right)./\boldsymbol{b}_\mathrm{t}\left(\theta_k\right), \tag{33}$$

where ./ denotes the element-wise division. Based on the error between the theoretical value and the estimated value of $\boldsymbol{c}_\mathrm{t}\left(r_k\right)$ under natural logarithm, least squares (LS) method is used for range estimation as

$$\begin{aligned}\hat{r}_k&=\arg\min_{r_k}||\ln\hat{\boldsymbol{c}}_\mathrm{t}\left(r_k\right)-\ln(\hat{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)./\hat{\boldsymbol{b}}_\mathrm{t}\left(\theta_k\right))||_2\\&=\arg\min_{r_k}||\boldsymbol{p}r_k-\boldsymbol{h}(\theta_k,r_k)||_2,\end{aligned} \tag{34}$$

where

$$\begin{aligned}\hat{\boldsymbol{b}}_\mathrm{t}\left(\theta_k\right)&=[e^{j2\pi\alpha_0 d\sin\left(\hat{\theta}_k\right)/\lambda},\cdots,e^{j2\pi\alpha_{M-1}d\sin\left(\hat{\theta}_k\right)/\lambda}]^T,\\ \boldsymbol{p}&=[-j4\pi\alpha_0\Delta f/c,\cdots,-j4\pi\alpha_{0M-1}\Delta f/c]^T,\\ \boldsymbol{h}(\theta_k,r_k)&=\ln(\hat{\boldsymbol{a}}_\mathrm{t}\left(\theta_k,r_k\right)./\hat{\boldsymbol{b}}_\mathrm{t}\left(\theta_k\right)).\end{aligned}$$

The closed-form solution can be obtained as

$$\hat{r}_k=\frac{\boldsymbol{p}^H\boldsymbol{h}(\theta_k,r_k)+\boldsymbol{h}(\theta_k,r_k)^H\boldsymbol{p}}{2\boldsymbol{p}^H\boldsymbol{p}},\ (k=0,1,\cdots,K-1). \tag{35}$$

C. Range estimation by search method

The proposed LS range estimation method requires low computational complexity, but better range estimation performance can by achieved by the 1D searches for the range estimation at the cost of increased computational complexity.

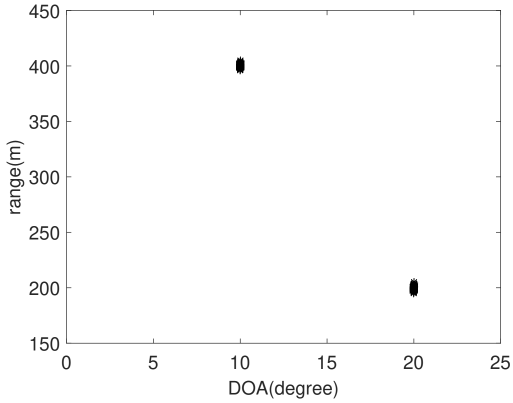

Fig. 2. Scatter plot of the DOA and range estimates

For each DOA estimate $\hat{\theta}_k, k = 1, 2, \cdots, K$, search function for the corresponding range parameter can be established based on (25) as

$$F(r) = \frac{1}{\boldsymbol{a}_{\mathrm{t}}(\hat{\theta}_k, r)^H \mathbf{Q}(\hat{\theta}_k) \boldsymbol{a}_{\mathrm{t}}(\hat{\theta}_k, r)}, \quad (36)$$

and the range estimate $\hat{r}_k, k = 1, 2, \cdots, K$ can be obtained from the maximum peak point of (36). Note that this method requires K times 1D searches, which require higher computational cost, for better estimation performance.

IV. Simulation Results

In this section, the effectiveness of the proposed RD MUSIC algorithm is verified through computer simulations with various conditions comparing with other algorithms. The performance of the algorithms is evaluated by root-mean-square error (RMSE) of the DOA estimation and the range estimation. For simplicity, we assume a co-located FDA MIMO radar, where the transmitting and receiving arrays share the same physical antennas. f_0 and Δf are set to be 10GHz and 5kHz, respectively. Two far-field targets with DOA and range parameters of $(\theta_0, r_0) = (10°, 400\mathrm{m})$ and $(\theta_1, r_1) = (20°, 200\mathrm{m})$ are assumed.

Fig. 2 shows the scatter plot of the DOA and range estimates pairs obtained by the proposed algorithm, where array structure is 1-level nest array (NA) [13] with the number of antenna elements $M = N = 10$. Signal-to-noise ratio (SNR) and the number of snapshots are set to be 20dB and $L = 100$, respectively. Search step for DOA estimates is $\Delta_\theta = 0.01°$ and range estimates are obtained by the closed-form LS solution. 100 independent simulation trials are performed, and each estimation result is shown as a point in Fig. 2. From the figure, it can be seen that all estimation results are close to the true points, which demonstrates the effectiveness of the proposed algorithm.

Figs. 3 and 4 demonstrate the RMSE performance of DOA and range estimation by the proposed and conventional methods using ULA with $M = N = 10$, where $\Phi_t = \Phi_r = \{0d, 1d, \ldots, 9d\}$. The number of snapshot of the received signal is $L = 10$. Search steps for DOA and range estimates are $\Delta_\theta = 0.01°$ and $\Delta_{\mathrm{r}} = 0.1$ m, respectively, for the algorithms which require peak search. Yellow line marked with cross is the performance of the conventional RD MUSIC algorithm [12], which require separate DOA and range search for each target. Red line marked with square is the performance of ANM algorithm [8], obtaining the covariance matrix by solving a SDP problem and apply ESPRIT for estimates. From Fig. 3, we can see that the DOA estimation performance of the proposed algorithm is better than that of the conventional RD MUSIC algorithm under low SNR region, and is comparable to the ANM ESPRIT algorithm under high SNR region. Moreover, Fig. 4 shows that the proposed low-complexity LS range estimation method can achieve comparable performance to the ANM ESPRIT algorithm, and that the proposed 1D search method for the range estimation can outperform the conventional RD MUSIC algorithm.

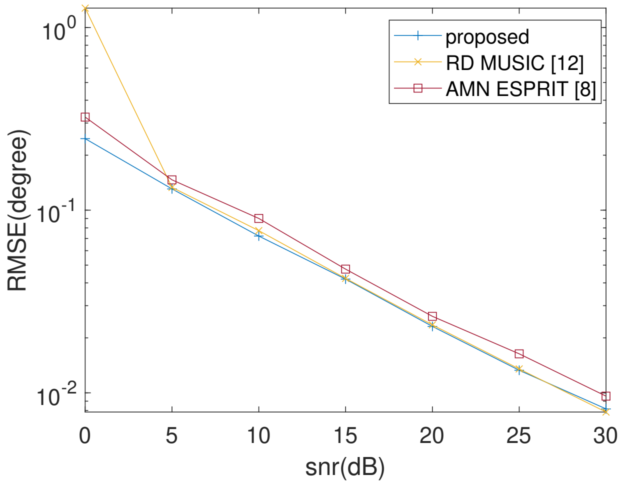

Fig. 3. RMSE of DOA estimates (ULA)

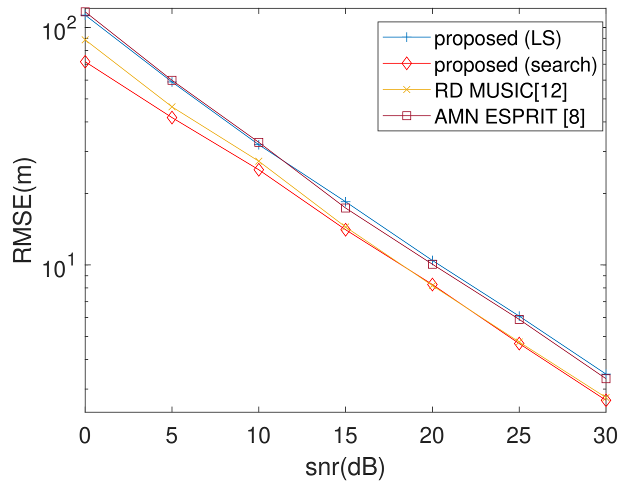

Fig. 4. RMSE of range estimates (ULA)

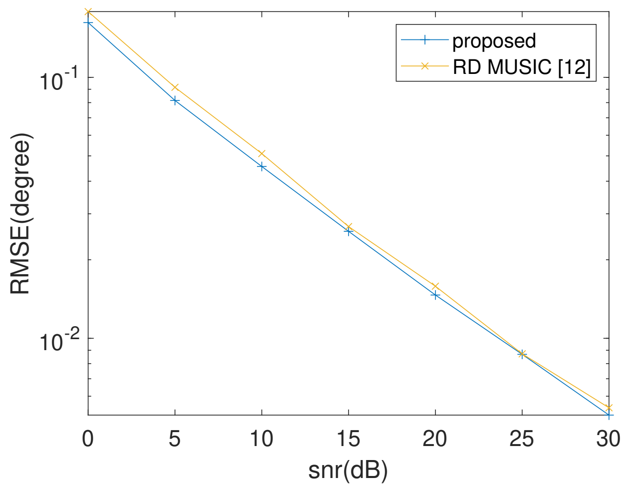

Fig. 5. RMSE of DOA estimates (1-level NA)

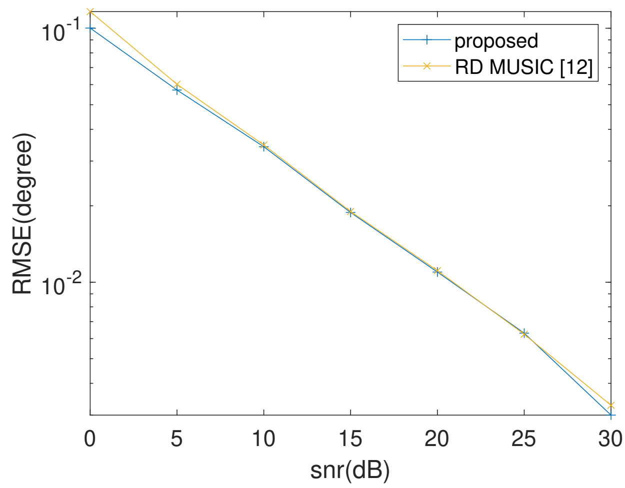

Fig. 7. RMSE of range estimates (2-level NA)

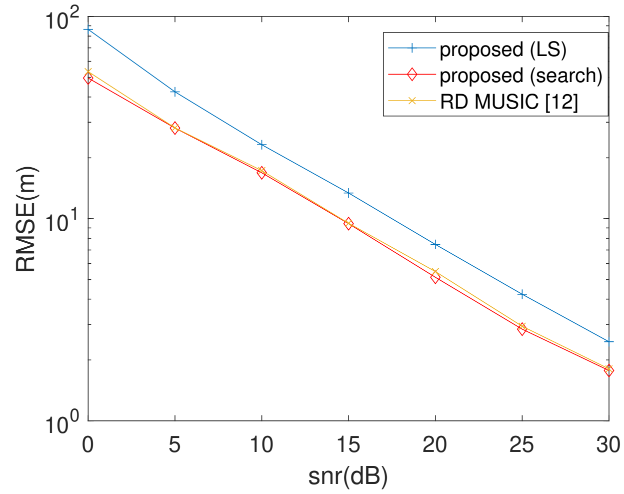

Fig. 6. RMSE of range estimates (1-level NA)

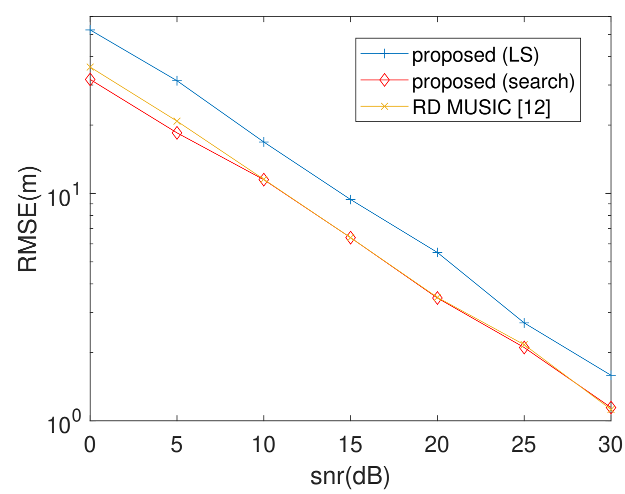

Fig. 8. RMSE of DOA estimates (2-level NA)

Figs. 5 and 6 illustrate the RMSE performance of DOA and range estimation by the proposed and conventional methods using 1-level NA with $M = N = 10$, where $\Phi_t = \Phi_r = \{0d, 1d, \dots, 8d, 17d\}$ and the number of snapshot of the received signal is $L = 10$. Search steps are the same as the previous case. Note that the performance of the ANM approach [8] is not shown here, because it requires the transmitting array to be ULA. From the results, we can see that the DOA estimation performance of the proposed algorithm is better than that of the conventional RD MUSIC algorithm in [12]. As for the range estimation performance, the proposed range estimation method with 1D search achieves the best performance for all SNR.

Figs. 7 and 8 show the RMSE performance for the case with 2-level NA with $M = N = 10$, where $\Phi_t = \Phi_r = \{0d, 1d, \dots, 7d, 15d, 23d\}$. Search steps are the same as the previous cases. Compared with the case of ULA or 1-level NA, it can be found that the RMSE performance of both DOA and range estimation is greatly improved. This is because, as the virtual array aperture increases, the estimation accuracy of the DOA improves, which indirectly leads to the improvement of the range estimation performance.

Finally, Figs. 9 and 10 show the RMSE performance of the DOA and range estimation for the case with co-prime array (CA) [14], which is composed by two sub-arrays given by $\Phi_1 = \{0d, 6d, 12d, 18d, 24d\}$ and $\Phi_2 = \{0d, 5d, 10d, 15d, 20d, 25d\}$. All the physical antennas are used for transmitting array and receiving array, written as $\Phi_t = \Phi_r = \Phi_1 \cup \Phi_2$, thus $M = N = 10$. Search steps are the same as the previous cases. From the results, the estimation performance for the case with CA is close to that with 2-level NA, verifying the effectiveness of the proposed algorithm on sparse arrays. Besides, the RMSE of the proposed algorithm can also outperform that of the conventional RD

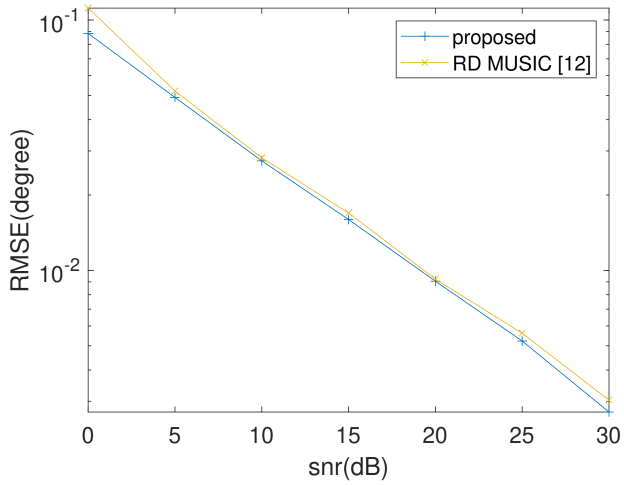

Fig. 9. RMSE of range estimates (CA)

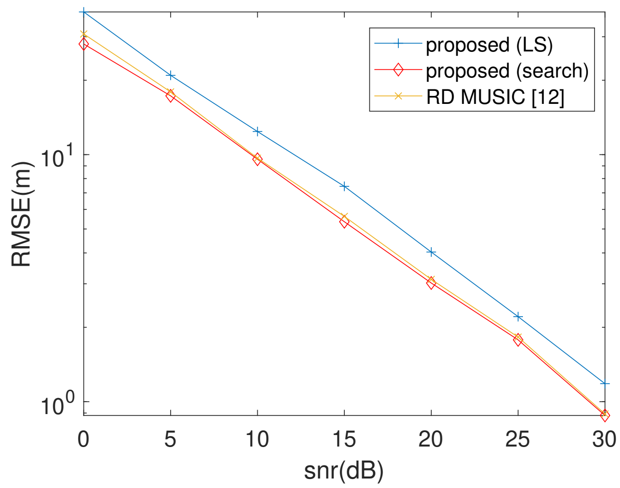

Fig. 10. RMSE of DOA estimates (CA)

MUSIC algorithm regardless of the array configuration.

V. Conclusions

In this paper, an improved RD MUSIC algorithm is proposed for FDA MIMO radar, which can be applied for arbitrary linear array. The proposed approach reshapes the MUSIC spectrum function and transforms the peak search operation on it into the minimization problem with the ℓ_2-norm constraint. After some manipulations, we get the search function only about DOA, and perform a 1D search to obtain DOA estimates. Finally, we use the LS method to get the closed-form solution for the range estimates. In addition, 1D search about range can perform better than the LS method at the cost of increased computational complexity. Computer simulation results prove that the algorithm is effective regardless of the array configuration.

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